

所 別： 電機工程技術研究所
學程別：

組別： 計算機組

科目： 離散數學

- (1) 8% Consider the problem of dealing a poker hand out of a deck of 52 cards.
- (a) What is the probability of getting four aces?
 - (b) What is the probability of getting four hearts and one club?
- (2) 10% Suppose we have a crooked die such that the probability of getting 1 or 2 is two times greater than that of getting 5 and the probability of getting 5 is three times greater than that of getting 3, 4 or 6. After rolling such a die once,
- (a) What is the probability of getting an even number?
 - (b) What is the probability of getting a number which is greater than 4?
- (3) 10% Let R be a binary relation on the set of all positive integers such that $R = \{(a, b) | a - b \text{ is an even positive integer}\}$
- (a) Is R reflexive? Symmetric? Antisymmetric? Transitive? A partial ordering relation?
 - (b) Repeat part (a), if $R = \{(a, b) | a = b^2\}$.
- (4) 16% Let f be the numeric function $(1 \times 0, 2 \times 1, \dots, i \times (i-1), \dots)$.
- (a) Determine the generating function of the numeric function f .
 - (b) Apply (a), to determine the sum $\sum_{i=0}^n i(i-1)$.
- (5) 6% Let $(A, *)$ be a group.
- (a) Show that $(a * b)^{-1} = b^{-1} * a^{-1}$.
 - (b) Show that $(a_1 * a_2 \dots a_{r-1} * a_r)^{-1} = a_r^{-1} * a_{r-1}^{-1} \dots a_2^{-1} * a_1^{-1}$.

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(6)10% Show that

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5 + \dots + n(n+1)(n+2) = n(n+1)(n+2)(n+3) / 4$$

(7)10% Show that among $n+1$ arbitrarily chosen integers, there are two whose difference is divisible by n .

(8)10% Let A be a set with 6 distinct elements.

- (a) How many different binary relations on A are antisymmetric?
- (b) How many different binary relations on A are reflexive?

(9)10% An algebraic system $(B, *)$ is isomorphic to another system $(A, \#)$ if there exists a one-to-one and onto function f from A to B such that for all a_1 and a_2 in A , $f(a_1 \# a_2) = XXX$.

- (a) Give the details to replace XXX.
- (b) In what condition, should the word "isomorphic" be changed to "homomorphic"?

(10)10% Let $A = \{0, 1\}$, $\#$ and \oplus be binary operations on A . $\#$ and \oplus are defined as $j\#k=j$, $j\oplus k=0$ if $j=k$, $j\oplus k=1$ if $j \neq k$, where $j, k \in A$.

- (a) Is the algebraic system $(A, \#)$ a semigroup? A monoid? A group?
- (b) Is the algebraic system (A, \oplus) a semigroup? A monoid? A group?