

1. Solve the differential equation

$$x^2 y'' + xy' + 4y = \cos(2\ln x) \quad (20\%)$$

2. Let S be a regular, closed surface, and let \vec{r} denote the position vector of any point measured from the origin O . If the origin lies outside S ,

calculate $\iint_S \frac{\vec{r}}{r^3} \cdot \vec{n} \, dS,$

where r is the norm of \vec{r} , and \vec{n} is a unit normal of S . (20%)

3. Find $\sqrt{[A]}$, if $[A] = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ (20%)

4. Solve the boundary value problem

$$\frac{\partial^2 y}{\partial t^2} = 4 \frac{\partial^2 y}{\partial x^2} \quad (-\infty < x < \infty, t \geq 0)$$

with $y(x, 0) = 0$, and $\frac{\partial y}{\partial t}(x, 0) = \delta(x)$, a Dirac delta function. (20%)

5. Use the residue theorem to evaluate the integral

$$\int_0^{\infty} \frac{x^{1/3}}{x(x^2 + 1)} dx \quad (20\%)$$