共四題，每題25分，可不依序作答，但題號請務必標示清楚。解題時請註明所依據之定律或原理；自由體圖須簡明繪於答案卷上。若需自行定義代號或向量，亦請於圖上標示清楚。重力加速度 (g) 之值請以 9.81 m/s² 或 32.2 ft/s² 計算。

1. A solid sphere A with a radius of R has an initial linear velocity v_0 and an initial angular velocity ω_0, as shown in Fig. 1. Another solid sphere B of identical radius and mass is sitting at rest at a distance of S away from sphere A. Now, sphere A is traveling on a rough horizontal surface toward sphere B and is going to make direct central impact with sphere B. After the impact, sphere A is standing still at the spot. Assume that the friction between spheres is negligible and the impact is perfectly elastic. Denoting by μ_k the coefficient of kinetic friction between the sphere and the floor, please answer the following questions:

   (a). Determine the linear velocities, $(v_B)_f$, and angular velocity, $(\omega_B)_f$, of sphere B immediately after the impact? Please express them in terms of v_0 and ω_0.

   (b). If ω_0 is unknown and S is given, with other conditions unchanged, find ω_0 as function of S.

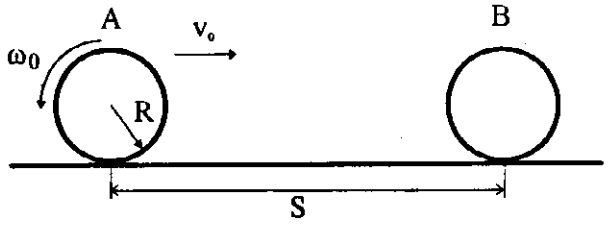
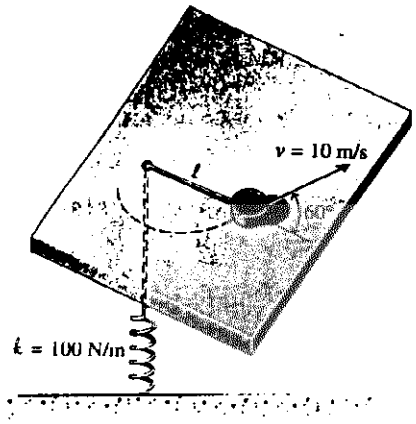


Fig. 1

2. A 0.6-kg mass slides on a smooth horizontal surface at the end of an inextensible string (Fig. 2). The other end of the string passes through a smooth hole in the surface and is attached to a spring having k =100 N/m. The spring is unstretched when $\ell = 0$. If v=10 m/s and $\ell = 0.5$ m at the instant shown,
   (a). determine the minimum and maximum values of ℓ in the resulting motion.
   (b). explain the physical phenomena when the string is at its maximum and minimum length.



3.The slotted disk sector rotates with a constant counterclockwise angular velocity $\omega = 4$ rad/s . Simultaneously the slotted arm OC oscillates about the line OB (fixed to the disk) so that θ changes at the constant rate of 2 rad/s except at the extremities of the oscillation during reversal of direction. Determine the magnitude of the total acceleration of the pin A when $\theta = 30°$ and θ is positive (clockwise). (Fig. 3)

$\frac{d}{dt}\cos\theta = -\dot{\theta}\sin\theta$; $\frac{d}{dt}\sin\theta = \dot{\theta}\cos\theta$; $\frac{d}{dt}\tan\theta = \dot{\theta}\sec^2\theta$; $\frac{d}{dt}\sec\theta = \dot{\theta}\sec\theta\tan\theta$

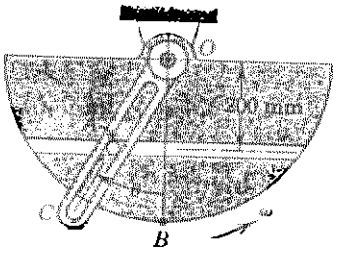


Fig. 3

4. The guide with the vertical slot is given a horizontal oscillatory motion according to $x = 100 \sin 2t$ where x is in millimeters and t is in seconds. The oscillation causes the pin P to move in the fixed slot whose shape is given by $y = \frac{x^2}{100} + \frac{0.2x}{x+10}$ with y also in millimeters. Find and locate the maximum value of the velocity of the pin between 0 and 1 second. Please solve the problem numerically by using a time step of 0.001 second. Write your computer program in one of the following languages: Fortran, Basic, or C. (Fig. 4)

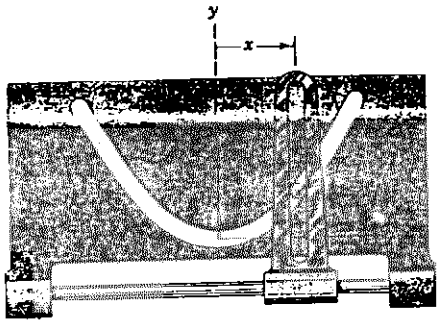


Fig. 4