所別：
學程別： 機械工程技術研究所 組別：熱流組 科目：流體力學

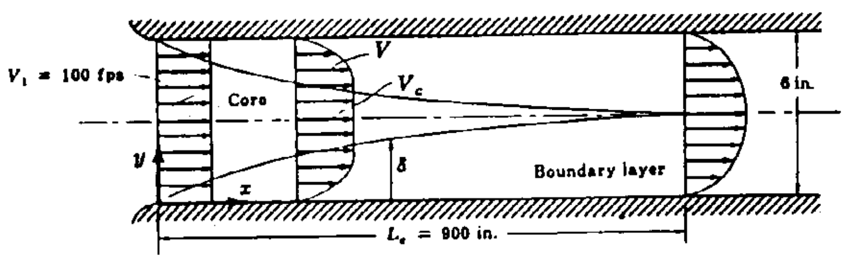
Note: Explain carefully every step you make in the following problems. No explanation gets you no points.

1. **(25 points)** Prove that $\frac{\partial p}{\partial x}=0$ for a viscous steady flow at a moderate to high Reynolds number over a flat plate at 0° angle of attack. The body force in the streaming (x) direction is negligible.

2. **(25 points)** Prove that $\frac{\partial v_z}{\partial z}=0$ for a steady, incompressible, and fully developed viscous flow inside a round pipe.

3. **(25 points)** Let $\frac{v_x}{V_\infty}=a+b\frac{y}{\delta}+c(\frac{y}{\delta})^2+d(\frac{y}{\delta})^3$. Determine the value of a, b, c, and d for a steady, incompressible, two-dimensional boundary layer flow over a flat plate at 0° angle of attack. Note that v_x is the local velocity of the flow along the plate, V_∞ is the free stream velocity along the plate, y is the direction perpendicular to the plate, and δ is the boundary layer thickness.

4. **(25 points)** Air flows between two large parallel walls as shown below. The velocity V_i is uniform and equal to 100 ft/s at the entrance. Downstream a distance of 900 inches the velocity varies over the entire width. The velocity varies in the boundary layer region according to $V = V_c(y/\delta)^{1/7}$ where $\delta = 0.1\sqrt{x}$ and δ and x are measured in inches. Determine the acceleration on the axis of symmetry for $0 \le x \le 900$ inches. Evaluate the acceleration at $x =$ 100 inches.



The following provides the fluid dynamics governing equations as a reference.

<u>Continuity Equation</u>:

Cartesian coordinate system:

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho v_x)+\frac{\partial}{\partial y}(\rho v_y)+\frac{\partial}{\partial z}(\rho v_z)=0$$

Cylindrical coordinate system:

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)=0$$

Navier-Stokes Equation (for a Newtonian fluid with constant ρ and μ)

Cartesian coordinate system:

x-component

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\rho g_x$$

y-component

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)+\rho g_y$$

z-component

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Cylindrical coordinate system:

r-component

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right]+\rho g_r$$

θ-component

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]+\rho g_\theta$$

z-component

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\rho g_z$$

Prandtl's two-dimensional boundary layer equation:

$$\rho\left(v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 v_x}{\partial y^2}$$