所　別：纖維及高分子工程技術研究所　組別：纖維甲、乙組、高分子組　科目：工程數學
學程別：

1. Find a general solution of the following equation.

$$4y'' + 8y' + 5y = e^{-x}\sec\frac{x}{2} \qquad (12\%)$$

2. Given that the matrix

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & b \\ 0 & b & c \end{bmatrix}$$

has $[1\ 0\ 1]^T$ as an eigenvector.

(a) Find the values of b and c. For these values of b and c calculate all the eigenvalues and corresponding eigenvectors of the matrix **A**.

(b) Calculate e^{At} (16%)

3. Express the quadratic form

$$Q = x_1^2 + 4x_1x_2 - 4x_1x_3 - 6x_2x_3 + \lambda(x_2^2 + x_3^2)$$

in the form $\mathbf{x}^T\mathbf{A}\mathbf{x}$, where $\mathbf{x} = [x_1\ x_2\ x_3]^T$ and **A** is a symmetric matrix. Hence determine the range of values of λ for which Q is positive definite. (10%)

4. (a) Determine the Fourier coefficients in the Fourier series

$$-3 = \sum_{n=1}^{\infty} B_n \sin n\pi x \qquad (0 < x < 1)$$

(b) If a function F(x) possesses the Fourier series

$$F(x) = \sum_{n=1}^{\infty} B_n \sin n\pi x$$

Obtain the solution of the following differential equation in the form of Fourier series.

$$u^{(4)}(x) + pu''(x) = F(x) \qquad 0 < x < 1$$

where $p > 0$ and $p \neq n^2\pi^2$ $(n = 1, 2, 3, \ldots)$.

Furthermore, use the result in (a) to obtain the solution in the special case when $F(x) = -3$. (12%)

5. Solve the following differential equations:

(a) $y(t) = t + \int_0^t y(\tau)\sin(t-\tau)d\tau$. (15%)

(b) Find the motion of the mass-spring system in Fig. 1 with mass 0.1 Kg, damping constant 0, spring constant 0.9 Kg/sec^2 and drivimg force cost − 4sint Newton, assuming zero initial displacement and velocity. For what frequency of the driving force would you get resonance ? (20%)

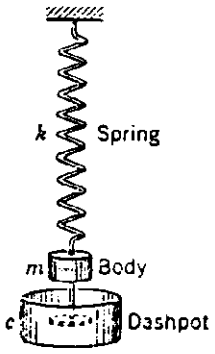

Fig. 1

(c) $xy' = y' + x^2\tan(y/x)$ (15%)