1. Convert the following argument into logical notation. Then establish the validity of the argument or show that it is invalid.

If there is a chance of rain or her white jacket is missing, then Grace will not go picnic.
When the temperature is over 30°C, there is no chance for rain.
Today the temperature is 32°C and Grace is wearing her white jacket.
Therefore Grace will go picnic today. (10%)

2 For $x, y \in R$, define an operation "o" on R as follows:

$x \circ y = x + y - x \cdot y$ where $+, -, \cdot$ denote usual addition, subtraction, and multiplication.

a). Show that the operation "o" is commutative and associative.

b). Find the identity element and indicate the inverse of each element (10%)

3. a). Define a sequence as:

$F_0 = 0$, $F_1 = 1$, and

$F_n = F_{n-1} + F_{n-2}$, for $n \in N$ with $n \geq 2$.

Show that

$$\sum_{i=0}^{n} (F_i)^2 = F_n \cdot F_{n+1} \quad \text{for } \forall n \in N \text{ by Mathematical Induction.}$$

b). Can you prove the statement

"$x^2 \geq 0$ for $\forall \cdot x \in R$" by Mathematical Induction? Why? (15%)

4. a). For $U = \{1,2,3\}$, let $A = P(U)$, the power set of U. Define the relation R on A by $(B, C) \in R$ if $B \subseteq C$. How many ordered pairs are there in the relation R?

b). Answer part (a) for $U = \{1,2,3,4\}$.

c). Generalize the results of parts (a) and (b). (15%)

5. A binary heap is a data structure that is an array object that can be viewed as a complete binary tree. Heaps satisfy a heap property: the value of a node is at most the value of its parent. Let A be an array with index $1...n$, and i be an index variable into A.

(a) Please express the following in terms of A and i:

i. $parent(i)$ – the index in A of the parent node of node i

ii. $left(i)$ – the index in A of the left sibling node of node i

iii. $right(i)$ – the index in A of the right sibling node of node i

iv. the heap property

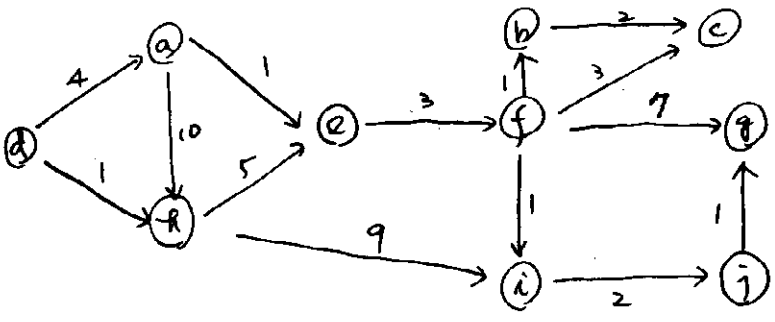
(b) Design a procedure called $heapify(A, i)$ that makes a tree rooted at node i into a heap, assuming both children of i, if any exists, are already heaps.

(c) Using $heapify(A, i)$, design an algorithm $buildheap(A)$ that makes an arbitrary array into a heap. (15分)

6. The Dijakstra's algorithm for finding single-source shortest paths is illustrated below, where $currDist()$, $predecessor()$ are integer array objects, and toBeChecked is a set variable for vertices.

```
DijkstraAlgorithm(weighted simple digraph, vertex first)
    for all vertices v
        currDist(v) = ∞;
    currDist(first) = 0;
    toBeChecked = all vertices;
    while toBeChecked not empty
        v = a vertex in toBeChecked with minimal currDist(v);
        remove v from toBeChecked;
        for all vertices u adjacent to v and in toBeChecked
            if currDist(u) > currDist(v)+ weight(edge(vu))
                currDist(u) = currDist(v)+ weight(edge(vu));
                predecessor(u) = v;
```

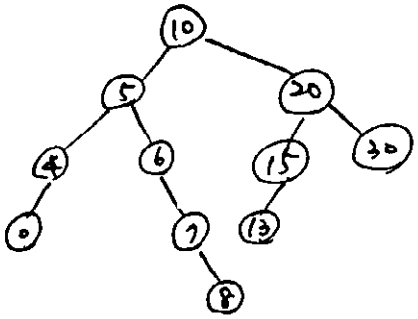
Do the Dijkstra's algorithm for the digraph below, with node d as the source. Please show all values of $curDist()$ in each iteration. (15分)



7. Tree traversal is the process of visiting, by calling the function *visit*(p), each node in the tree exactly once. Draw the resulting tree after pre-order, in-order, and post-order traversal of the following tree, if *visit*(p) to the node p is defined to be: (10分)

(a)
```
if ( p->left && p->key - p->left->key < 2 )
    p->left->key += 2;
```

(b)
```
if ( ! p->left )
    p->right = 0;
```



8. For a given function $g(n)$, denote by $O(g(n))$ the set of functions

$$\{f(n) : \exists c \geq 0, n_0 \geq 0, s.t. 0 \leq f(n) \leq cg(n), \forall n \geq n_0\}$$

Prove that

$$\frac{1}{2}n^2 - 3n \in O(g(n))$$

where $g(n) = n^2$. (10分)