

所別：電子工程技術研究所
學程別：

組別：系統組、元件與材料組

科目：工程數學

(1)(12%) Solve $x^3 y''' - 3x^2 y'' + 6xy' - 6y = x^4 e_n x$

(2)(12%) Let D be the region bounded by the hemisphere $x^2 + y^2 + (z-1)^2 = 9$
 $1 \leq z \leq 4$ and the plane $z=1$. Verify the divergence theorem if

$$\vec{F} = x\vec{i} + y\vec{j} + (z-1)\vec{k}$$

(3)(12%) Let $F(x,y)$ and $G(x,y)$ be scalar field in the plane, Let C be a simple closed piecewise-smooth curve in the plane enclosing a region D over which F,G and their first and second derivative are continuous, show that

$$\iint_D F \nabla^2 G dA = \oint_C \left[-F \frac{\partial G}{\partial y} dx + F \frac{\partial G}{\partial x} dy - \iint_D \nabla F \cdot \nabla G dA \right]$$

(4)(12%) Suppose $\{\phi_n(x)\}$ is an infinite orthogonal set of function on an interval $[a,b]$ and $f(x)$ is a function defined on the interval $[a,b]$ prove that

$$f(x) = \sum_{n=0}^{\infty} \frac{(f, \phi_n)}{\|\phi_n(x)\|^2} \phi_n(x) \text{ where } \|\phi_n(x)\|^2 = \int_a^b \phi_n^2 dx, (f, \phi_n) = \int_a^b f(x) \phi_n(x) dx$$

(5)(12%) $A = \begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$, find Q such that $Q^{-1}AQ$ is diagonal, where Q is 3x3 matrix.

(6)(12%) let $f(z)$ have a pole of order m at Z_0 , then

$$\text{Res}_{Z_0} f = \frac{1}{(m-1)!} \lim_{z \rightarrow Z_0} \frac{d^{m-1}}{dz^{m-1}} \left[(z - Z_0)^m f(z) \right]$$

(7)(12%) Evaluate the Cauchy principal value of $\int_{-\infty}^{\infty} \frac{\sin x}{x(x^2 - 2x + 2)} dx$

(8) (16%) Solve $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \quad (0 < x < 2, t > 0)$
 $y(0,t) = y(2,t) = 0 \quad (t > 0)$
 $y(x,0) = 0 \quad (0 < x < 2)$
 $\frac{\partial y}{\partial t}(x,0) = 2x \quad (0 < x < 2)$