國立臺灣科技大學
八十七學年度碩士班招生考試試題

所　別：工程技術研究所　　組別：　　科目：離散數學
學程別：資訊工程學程

Each of the following ten problems deserves 10 points. Note that the process to obtain your answer should be explained in detail.

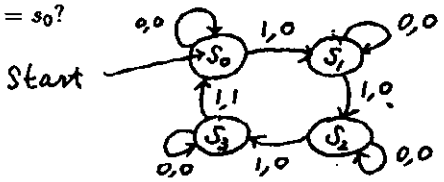
(1) The universe consists of all positive real numbers. Given a statement "Every positive real number has a unique multiplicative inverse", express this statement in symbolic form.

(2) For n a positive integer, prove that $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$.

(3) When does a positive integer n have exactly 15 positive divisors?

(4) Given $A = \{a, b, c, d\}$ and $B = \{x, y, z\}$, how many onto functions are there from A to B?

(5) Let M be the finite state machine as shown below and v be the transition function. How many distinct input strings x are there such that the length of x is 8 and $v(s_0, x) = s_0$?



(6) Draw the Hasse diagram for the set of positive integer divisors of the integer 30.

(7) Find the coefficient of x^{63} in $f(x) = (x^5 + x^8 + x^{11} + x^{14} + x^{17})^{10}$.

(8) Solve the relation $x_n - 3x_{n-1} = n$ for $n \geq 1$ and $x_0 = 1$.

(9) The connected undirected graph $G = (V, E)$ has 30 edges. What is the maximum value that $|V|$ can have?

(10) If a is a unit in ring R, prove that $-a$ is also a unit in R.