

國立臺灣科技大學
八十七學年度碩士班招生考試試題所 別：管理技術研究所
學程別：工業管理學程

組別：

科目：統計學

There are 4 problems in this exam. Show intermediate steps and formulas for partial credit. You must explain how you compute your results or answers for full credit.

1. (25 points)

Three cards are drawn without replacement from 12 cards (Jacks, Queens, and Kings) of an ordinary deck of 52 playing cards. Let X be the number of Kings selected and Y be the number of Jacks.

- (a) (5 points) Find the joint probability distribution of X and Y .
- (b) (5 points) Find $P\{X + Y \geq 2\}$.
- (c) (5 points) Find $E[X]$.
- (d) (5 points) Find the marginal distribution of Y .
- (e) (5 points) Find the conditional distribution of X , given that $Y=1$.

2. (25 points)

A company distributes boxes of a mixture of materials A, B and C. Suppose that the weight of each box is 1 kg, but the individual weights of the materials A, B and C vary from box to box. For a randomly selected box, let X and Y represent the weights of the material A and B, respectively and suppose that joint density function of X and Y is given as follows.

$$f(x, y) = \begin{cases} 24xy, & 0 \leq x \leq 1, 0 \leq y \leq 1, x + y \leq 1 \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) (5 points) Find the probability that the weight of material C is more than 0.5 kg in a given box.
- (b) (10 points) Find the probability that the weight of material B is less than 0.125 kg if the weight of material A is 0.75 kg in a box.
- (c) (10 points) Find the probability density function of the random variable $Z=X+Y$.

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3. An industrial engineer is interested in the outputs of two assembly lines. Independent tests for the two lines show the following results: (共 30%)

line A	line B
$n_A = 10$	$n_B = 16$
$\bar{X}_A = 14.5$	$\bar{X}_B = 11.3$
$S_A = 0.8$	$S_B = 0.7$

- Construct 95% Confidence interval on the true variances for line A and line B respectively. (10%)
- Construct 95% Confidence interval on the true means for line A and line B respectively. (10%)
- Test if variances for line A and line B are equal ($\alpha = .05$), and using the result to test if mean outputs for line A and line B are equal ($\alpha = .05$). (10%)

4. A production process is checked periodically by a quality control inspector. The inspector selects simple random sample of 36 products and computes the sample mean weights $\bar{X}$. (共 20%)

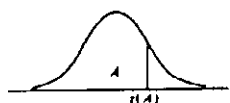
- If test results over a long period of time show that 10% of the $\bar{X}$ values are over 12.32 pounds and 9% are under 11.63 pounds, what are the mean and standard deviation for the population produced with this process? (10%)
- If overweight or underweight is a serious problem, and the production line should be shut down if either occurs. With a 0.05 level of significance, what is the rule for shutting the process down? (10%)

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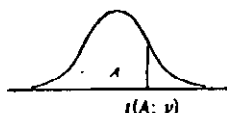
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Entry is area A under the standard normal curve from $-\infty$ to $z(A)$ 

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
10	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
11	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
12	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
13	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
14	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
15	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
16	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
17	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
18	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
19	.9712	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
20	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
21	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
22	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
23	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
24	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
25	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
26	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
27	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
28	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
29	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
30	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
31	.9990	.9991	.9991	.9992	.9992	.9992	.9992	.9992	.9993	.9993
32	.9993	.9993	.9994	.9994	.9994	.9994	.9995	.9995	.9995	.9995
33	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	.9997
34	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Selected Percentiles

Cumulative probability A	.90	.95	.975	.98	.99	.995	.999
$z(A)$	1.282	1.645	1.960	2.054	2.326	2.576	3.090

Entry is $t(A; v)$ where $P\{t(v) \leq t(A; v)\} = A$ 

v	A							A						
	.60	.70	.80	.85	.90	.95	.975	.98	.985	.99	.9925	.995	.9975	.9995
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303	4.819	5.643	6.965	9.925	14.089	31.598	31.598
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182	3.182	3.182	3.182	3.182	3.182	3.182	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776	2.776	2.776	2.776	2.776	2.776	2.776	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571	2.571	2.571	2.571	2.571	2.571	2.571	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447	2.447	2.447	2.447	2.447	2.447	2.447	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365	2.365	2.365	2.365	2.365	2.365	2.365	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306	2.306	2.306	2.306	2.306	2.306	2.306	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262	2.262	2.262	2.262	2.262	2.262	2.262	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228	2.228	2.228	2.228	2.228	2.228	2.228	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201	2.201	2.201	2.201	2.201	2.201	2.201	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179	2.179	2.179	2.179	2.179	2.179	2.179	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160	2.160	2.160	2.160	2.160	2.160	2.160	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145	2.145	2.145	2.145	2.145	2.145	2.145	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131	2.131	2.131	2.131	2.131	2.131	2.131	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120	2.120	2.120	2.120	2.120	2.120	2.120	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110	2.110	2.110	2.110	2.110	2.110	2.110	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101	2.101	2.101	2.101	2.101	2.101	2.101	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093	2.093	2.093	2.093	2.093	2.093	2.093	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086	2.086	2.086	2.086	2.086	2.086	2.086	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080	2.080	2.080	2.080	2.080	2.080	2.080	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074	2.074	2.074	2.074	2.074	2.074	2.074	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069	2.069	2.069	2.069	2.069	2.069	2.069	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064	2.064	2.064	2.064	2.064	2.064	2.064	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060	2.060	2.060	2.060	2.060	2.060	2.060	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056	2.056	2.056	2.056	2.056	2.056	2.056	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052	2.052	2.052	2.052	2.052	2.052	2.052	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048	2.048	2.048	2.048	2.048	2.048	2.048	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045	2.045	2.045	2.045	2.045	2.045	2.045	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042	2.042	2.042	2.042	2.042	2.042	2.042	2.042
40	0.255	0.529	0.853	1.050	1.303	1.684	2.021	2.021	2.021	2.021	2.021	2.021	2.021	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980	1.980	1.980	1.980	1.980	1.980	1.980	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960	1.960	1.960	1.960	1.960	1.960	1.960	1.960

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrika* 32 (1941), pp. 198-199.

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v ₁	Degrees of Freedom for the Numerator (v ₂)																			
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞	
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	997.2	1001	1006	1010	1014	1018	
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50	
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.93	
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.28	
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.04	
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.88	
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.18	
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.71	
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.37	
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.12	
11	6.72	5.26	4.63	4.26	4.04	3.86	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.92	
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.77	
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.64	
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.85	2.79	2.73	2.67	2.61	2.55	2.53	
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.44	
16	6.12	4.69	4.07	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.36	
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.30	
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.24	
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.19	
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.15	
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.10	
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.07	
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.99	
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.97	
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.95	
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.93	
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.90	
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.88	
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.86	
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.84	
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.69	
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.54	
120	5.15	3.80	3.21	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.40	
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.24	