國立臺灣科技大學
八十七學年度碩士班招生考試試題

所　別：管理技術研究所　　組別：甲組　　科目：離散數學
學程別：資訊管理學程

1. For any $n>0$, show that $C(n,0)+3C(n,1)+3^2C(n,2)+\ldots+3^nC(n,n)=4^n$. (10%)

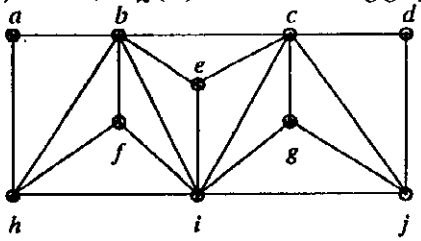
2. Define the connective $p\downarrow q \Leftrightarrow \neg(p\vee q)$. Represent the following statements using only $p\downarrow q$: (8% each)
   (a) $p\rightarrow q$
   (b) $p\leftrightarrow q$

3. Determine the number of integer solutions of the equation:
   $x_1+x_2+\ldots+x_6=25$, *where* $x_i\geq 2$ *for* $1\leq i\leq 6$. (10%)

4. Given two non-empty sets A and B, where $|A|=m$, $|B|=n$.
   Answer the following: (7% each)
   (a) Assuming $n\geq m$, how many possible injective functions are there from A to B?
   (b) Assuming $m\geq n$, how many possible surjective functions are there from A to B?

5. Let $G=(V,E)$ be a loop-free connected planar graph with $|V|=v$, $|E|=e>2$, and r regions.
   (a) Prove that $3r\leq 2e$ and $e\leq 3v-6$. (7%)
   (b) Prove that K_5 and $K_{3,3}$ are not planar graphs. (8%)

6. Let $G=(V,E)$ be an undirected graph, a proper coloring of G occurs when we color the vertices of G so that if $\{a,b\}$ is an edge in G, then a and b are colored with different colors. (Hence adjacent vertices have different colors.) The minimum number of colors needed to properly color G is called the chromatic number of G and is written $\chi(G)$.
   (a) Determine $\chi(G)$ for the following graph. (5%)



   (b) Let λ be the number of colors that are available for properly coloring the vertices of G. A polynomial function $P(G,\lambda)$, in the variable λ, is called the chromatic polynomial of G if $P(G,\lambda)$=the different number of ways to properly color G with λ colors. Show that for a path G with n vertices, $P(G,\lambda)=\lambda(\lambda-1)^{n-1}$ (5%)
   (c) For $e=\{a,b\}\in E$, let G_e denote the subgraph of G obtained by deleting e from G without removing vertices a and b. From G_e a second graph of G is obtained by coalescing the vertices a and b (i.e., a and b are combined into one vertex). This second subgraph is denoted by G_e'. Prove that if $G=(V,E)$ is a connected graph and $e\in E$, then $P(G_e,\lambda)=P(G,\lambda)+P(G_e',\lambda)$. (10%)

7. Construct a six vertices, ten edges loop-free and positively weighted connected undirected graph where at least two minimal spanning trees (i.e., two spanning trees with the same minimal weight) exist. (5%) Under what conditions that this graph has only one minimal spanning tree. (5%) Prove it. (5%)