

國立臺灣科技大學
八十七學年度碩士班招生考試試題

所 別：管理技術研究所
學程別：資訊管理學程

組別：丙組

科目：作業研究

Operations Research (Mar. 1998)

1.(5%) We have k urns, labeled $1, 2, \dots, k$, and n identical (indistinguishable) balls. In how many ways can these balls be distributed in k urns?

2.(5%) Two people take turns rolling a die. Peter rolls first, then Paul, then Peter again, and so on. The winner is the first to roll a six. What is the probability that Peter wins?

3.(5%) Let X be a continuous random variable with cdf F . Assume that F is strictly increasing. After observing the value of X we compute $Y = F(X)$. Find the cdf of the random variable Y .

4.(15%) When using estimator $\hat{\theta}$ to estimate parameter θ , why is it so important to assess the variability of $\hat{\theta}$? How would you assess the error $|\hat{\theta} - \theta|$? What is the role played by sampling distribution of $\hat{\theta}$ in statistics? Why do we need to learn probability before statistics?

5.(20%) Given a matrix

$$A = \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix},$$

what is the limit of A^k as $k \rightarrow \infty$?

6.(15%) Find the projection (in terms of vector) when projecting the vector

$$\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

onto

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}.$$

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7.(10%) Data from the progression of college students at NTUST are summarized in the following transition probabilities.

	Graduate	Dropout	Freshmen	Sophomore	Junior	senior
Graduate	1.00	0.00	0.00	0.00	0.00	0.00
Dropout	0.00	1.00	0.00	0.00	0.00	0.00
Freshmen	0.00	0.25	0.10	0.65	0.00	0.00
Sophomore	0.00	0.20	0.00	0.12	0.68	0.00
Junior	0.00	0.08	0.00	0.00	0.06	0.86
Senior	0.92	0.02	0.00	0.00	0.00	0.06

In an address to the incoming class of 800 freshmen, the dean mentioned that about 50% of the freshmen present today will not make it to the graduation day. Does your Markov process analysis support the dean's statement? Explain.

8.(10%) Pacific Publishing Company publishes college textbooks. The company operates a free telephone call where potential adopters can ask questions or place orders. Currently 3 extension lines are used, with 3 representatives handling the calls. Calls occurring when all extension lines are busy receive a busy tone and no waiting is allowed. Each representative handles, on the average, 10 calls per hour.

(a) If the mean arrival rate is 20 calls per hour, what percentage of the calls receive a busy tone?

(b) What is the average number of extensions that will be busy?

9.(15%) A company produces three products requiring resources J and K . Management wants to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, 3 units of resource J , and 1 unit of resource K are required. For each unit of product 2, 3 units of resource J , and 4 units of resource K are required. For each unit of product 3, 2 units of resource J , and 1 unit of resource K are required. The company has 300 units of resource J , and 400 units of resource K . Each unit of product 1 gives a profit of \$2, each unit of product 2 gives a profit of \$1, and each unit of product 3, up to 50 units, gives a profit of \$3. Any excess over 50 units of product 3 brings no profit.

(a) Formulate a linear programming model for this problem.

(b) Solve this model using simplex method.

(c) If each unit of product 1 gives a profit of \$3, will the original solution remain optimal? Explain.

(d) Write the dual problem for the linear programming model of part (a) and solve the dual problem to verify your answer in (b).