

國立臺灣科技大學
八十七學年度碩士班招生考試試題

所 別： 化學工程技術研究所
學程別：

組別：

科目：工程數學

1. (10%) Find the general solution of $x^2 y'' - xy' + y = 6x$ for $x > 0$

2. (15%) Solve the following initial value problem.

$$y'' + 4ty' - 4y = 0; \quad y(0) = 0, y'(0) = -7$$

3. (15%) Find the general solution of

$$Y' = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} Y$$

4. (10%) Find the equations of the tangent plane and normal line to the surface $z^2 = x^2 - y^2$ at the point $(1, 1, 0)$

5. (20%) Please solve the following partial differential equation of the temperature $T(r, z)$ distribution in a cylindrical solid

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0$$

$$\text{with } T(r, 0) = 0$$

$$T(R, z) = 0$$

$$T(r, L) = f(r)$$

where R is the radius and L is the length of the cylindrical solid.

6. (20%) Please derive the equation of continuity

$$\nabla \cdot \rho \mathbf{v} + \frac{\partial \rho}{\partial t} = 0$$

from the control volume expression for the conservation of mass

$$\iint \rho(\mathbf{v} \cdot \mathbf{n}) dA + \frac{\partial}{\partial t} \iiint \rho dV = 0$$

where ρ is the density, \mathbf{v} is the velocity, ∇ is the gradient, \mathbf{n} is the normal vector of dA , V is the volume, and t is the time.

7. (10%) Please solve the following differential equation

$$(2 + x^2 y) \frac{dy}{dx} + xy^2 = 0; y(1) = 2$$