國立臺灣科技大學
八十七學年度碩士班招生考試試題

所 別：電機工程技術研究所 組別：計算機組 科目：離散數學
學程別：

(1) 9% Consider the problem of dealing a poker hand (5 cards) out of a deck of 52 cards.
(a) What is the probability of getting two pairs?
(b) What is the probability of getting a full house?
(c) What is the probability of getting four hearts and one club?

(2) 9% Suppose we have a crooked die such that the probability of getting 1 or 2 is three times greater than that of getting 6, the probability of getting 6 is two times greater than that of getting 3 and the probability of getting 3 is four times greater than that of getting 4 or 5. After rolling such a die once,
(a) What is the probability of getting an odd number?
(b) What is the probability of getting a number which is greater than 4?
(c) Given that an odd number appeared, what is the probability of getting a number which is greater than 4?

(3) 10% Find the number of distinct strings of length 3 that are made up blue beads and yellow beads. The two ends of a string are not marked, and two strings are, therefore, indistinguishable if interchanging the ends of one will yield the other. How about the number of distinct strings of length 4?

(4) 10% The Tower of Hanoi problem. r rectangles of tapering sizes are slipped onto a peg with the largest rectangle at the bottom. These rectangles are to be transferred one at a time onto another peg, and there is a third peg available on which rectangles can be left temporarily. If, during the course of transferring the rectangles, no rectangles may ever be placed on top of a smaller one, in how many moves can these rectangles be transferred with their relative positions unchanged? (Hint, by generating function.)

(5) 12% Let (I, ×) be a group, where I is the set of all integers and × is the ordinary multiplication operation of integers. Also let E be the set of all even integers.
(a) Show that (E, ×) is a normal subgroup of (I, ×).
(b) Based on (E, ×), find a homomorphic image of (I, ×).

(6) 10% Show that for any integer n, $(11)^{n+2} + (12)^{2n+1}$ is divisible by 133.

(7) 10% Let R be a binary relation on the set of all strings of 0s and 1s such that R = { (a,b) | a and b are strings and the length of a is equal to or greater than b}. Is R antisymmetric? Transitive? An equivalent relation? A partial ordering relation? A compatible relation?

(8) 10% Let T be a tree with 100 edges. The removal of a certain edge from T yields two disjoint trees T1 and T2. Given that the number of vertices in T1 equals the sum of the number of edges and the number of vertices in T2, determine the number of edges

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in T1 and T2.

(9) 10% Give an example of a lattice that isn't distributive. Then, show that it is indeed the one required.

(10) 10% Let A={a, b}, (A, *) be a semigroup, and b*b = a. The algebraic system is shown in the following table. List out all possible values for the blanks in the table to satisfy the given condition.

*	a	b
a		
b		a