國立臺灣科技大學
八十七學年度碩士班招生考試試題

所別：學程別：電子工程技術研究所　組別：計算機組　科目：離散數學與資料結構

1. (a) Write the negation of
" For every x, if $x \in A$, then $x \in B$."

(b) Write the contrapositive of
"$(x,y) \in R$ and $(y,x) \in R$ imply that $x = y$."

(c) Prove or disprove:
"If $(q \wedge r) \rightarrow p$ and $q \rightarrow \neg r$, then p."

(10%)

2. Let R_1 be a relation on {1,2,3,4} whose Boolean matrix is

$$R_1 : \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

(a) Determine whether R_1 is an equivalence relation, or a partial order, or neither. Explain your answer.

(b) If R_1 is an equivalence relation, then find the corresponding equivalence classes.
If R_1 is a partial order, then extend R_1 to a total order that preserves the original ordering.
If R_1 is neither an equivalence relation nor a partial order, then find the smallest equivalence relation containing R_1.

(10%)

3. Let R_2 be a relation on {a,b,c,d,e} whose Boolean matrix is

$$R_2 : \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(a) Determine whether R_2 is an equivalence relation, or a partial order, or neither. Explain your answer.

(b) If R_2 is an equivalence relation, then find the corresponding equivalence classes.
If R_2 is a partial order, then extend R_2 to a total order that preserves the original ordering.
If R_2 is neither an equivalence relation nor a partial order, then find the smallest equivalence relation containing R_2.

(10%)

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4. Given group $(G, \bullet)$.
The subgroup of $(G, \bullet)$ generated by A, denoted by $\langle A \rangle$, is defined as:
(B) $A \subseteq \langle A \rangle$.
(R_1) If $g, h \in \langle A \rangle$, then $g \bullet h \in \langle A \rangle$.
(R_2) If $g \in \langle A \rangle$, then $g^{-1} \in \langle A \rangle$.

(a) Consider the group $(Z(12), +_{12})$.
Find $\langle\{3\}\rangle$ in $(Z(12), +_{12})$. (i.e., find the subgroup of $(Z(12), +_{12})$ generated by $\{3\}$).

(b) Determine whether $(\langle\{3\}\rangle, +_{12})$ is isomorphic to $(Z(4), +_4)$ or not. Why?

(c) Let H be a subgroup of a group $(G, \bullet)$. A <u>left coset</u> of H in G is a subset of the form $g \bullet H = \{ g \bullet h : h \in H \}$.
Find the left coset $1 +_{12} \langle\{3\}\rangle$ of $\langle\{3\}\rangle$ in $(Z(12), +_{12})$.

(d) Show that the left cosets of $\langle\{3\}\rangle$ in $(Z(12), +_{12})$ form a partition of $Z(12)$.

(20%)

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5. For each of the following six program fragments, give an analysis of the running time in Big-Oh notation:

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```
(1)  Sum = 0;
     for( i = 0; i < N; i++ )
             Sum++;

(2)  Sum = 0;
     for( i = 0; i < N; i++ )
         for( j = 0; j < N; j++ )
             Sum++;

(3)  Sum = 0;
     for( i = 0; i < N; i++ )
         for( j = 0; j < N * N; j++ )
             Sum++;

(4)  Sum = 0;
     for( i = 0; i < N; i++ )
         for( j = 0; j < i; j++ )
             Sum++;

(5)  Sum = 0;
     for( i = 0; i < N; i++ )
         for( j = 0; j < i * i; j++ )
             for( k = 0; k < j; k++ )
                 Sum++;

(6)  Sum = 0;
     for( i = 1; i < N; i++ )
         for( j = 1; j < i * i; j++ )
             if( j % i == 0 )
                 for( k = 0; k < j; k++ )
                     Sum++;
```

6. A majority element in an array, A, of size N, is an element that appears more than (N/2) times. Here is a sketch of an algorithm to solve the problem:

Step1: find a candidate majority element.

To find a candidate in the array, A, form a second array, B. Then compare A[1] and A[2]. If they are equal, add one of these to B; otherwise do nothing. Then compare A[3] and A[4]. Again if they are equal, add one of these to B; otherwise, do nothing. Continue in this fashion until the entire array A is read. Then recursively find a candidate for B; etc.

Step 2: determine if the candidate found is actually the majority.

Do a sequential search through the array A to verify.

(a) How do you handle the case when there is no majority element at all?

(b) What is the running time of the algorithm?

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（命題用紙）

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7. A binary search tree supports *search, minimum, successor*, etc, operations.
(a) Give the pseudo-code for the *minimum(x)* operation, given x as a pointer to a node of the tree.
(b) Give the pseudo-code for the successor*(x)* operation, given x as a pointer to a node of the tree, using *minimum(x)* as a subroutine.
(c) Argue that if a node in a binary search tree has two children, then its successor has no left child and its predecessor has no right child.

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8. Given input {371, 323, 173, 199, 344, 679, 989} to be hashed into a table of size 10, and a hash function $h(x) = x \bmod 10$. Show the resulting hash table contents when each of the following hashing strategies are used.
(a) open addressing with linear probing,
(b) open addressing with quadratic probing, $h(x, i) = h(x) + i + i^2$,
(c) open addressing with second hash function $h'(x) = (x \bmod 9) + 1$.

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