國立臺灣科技大學
八十七學年度碩士班招生考試試題

所 別：
學程別：電子工程技術研究所　　組別：系統組　　科目：工程數學

1. Let curve C1 be an oblique trajectory of curve C2 with intersect angle α between C1 and C2, $0<\alpha\leq\pi/2$ as shown in fig.1.

   (1) (4%) Assume the slopes of C1 and C2 at (x,y) are m1 and m2 respectively. Show that

   $$m2=\frac{m1\ +\ \tan\alpha}{1-m1*\tan\alpha}$$

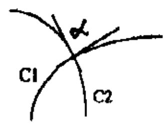

fig.1

   (2) (4%) Assume the curves C1 and C2 are orthogonal trajectories mutually. Show that $m1*m2=-1$

   (3) (4%) Find the orthogonal trajectory of the circles with radius R

   $$x^2+y^2=R^2$$

2. Consider the RC circuit shown in fig.2 with zero charge in the capacitor at $t=0$.

   (1) (4%) Find the differential equation of this circuit in terms of vi and vo.

   (2) (4%) Find the Laplace transform H(S) = Vo(S)/Vi(S) of this circuit.

   (3) (6%) Assume $vi(t)=A_1\cos\omega_1 t+A_2\cos\omega_2 t$. Find vo(t).

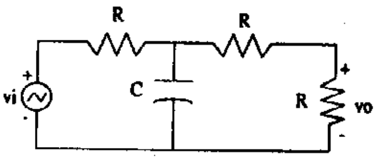

fig.2

3. (10%) Solve the initial value problem using the Laplace Transform

   $$y''+9y=f(t)\quad ;\quad y(0)=y'(0)=0\quad ,\quad \text{in which}$$

   $$f(t)=\begin{cases}0 & \text{if}\quad 0\leq t<4\\ t & \text{if}\quad t>4\end{cases}$$

4. Prove the following Fourier Transform identities of real functions x(t), y(t)

   (1)(8%) $$\int_{-\infty}^{\infty}\left\{\int_{-\infty}^{t}x(\tau)d\tau\right\}e^{-j2\pi ft}dt=\frac{X(f)}{j2\pi f}+\frac{X(0)\delta(f)}{2}$$

   (2)(6%) $$\int_{-\infty}^{\infty}x(t)y(t)dt=\int_{-\infty}^{\infty}X(f)Y^{*}(f)df$$

命題請勿超出線外

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5.In each case use the Gram-Schmidt formula to obtain an orthonormal set from the given linearly independent set.
(a)(5%){(4,0),(2,1)}
(b)(5%){(2,0,1),(1,1,1),(-2,0,3)}.

6.In each case determine the dimension of the subspace S of R^4.
(a)(5%)S consists of all vectors (x,y,-y,x+y,z),where x,y,z $\in$ R.
(b)(5%)S consists all vectors parallel to the plane 4x+y-z=0 in R^3.

7. Prove each of the following statements.
(a)(5%)If λ is an eigenvalue of A with a corresponding eigenvector **e**, then λ^n is an eigenvalue of A^n, with the same eigenvector **e**, for any integer n.
(b)(10%)If A is symmetric, then eigenvectors corresponding to distinct eigenvalues are orthogonal.

8.S consists of all points on cos(x)+sin(x)+z=3 and **F**= $e^x\cos(y)$**i**- $e^x\sin(y)$**j**.
(a)(5%)Find the equation of the tangent plane to surface S at (0,0,2).
(b)(10%)Evaluate $\int_C \mathbf{F}\cdot d\mathbf{R}$ for C any path from (0,0) to (2,45°).