

國立臺灣科技大學
八十七學年度碩士班招生考試試題

所 別： 電子工程技術研究所
學 程 別：

組 別： 元件與材料組

科 目： 工程數學

(一) Evaluate $\oint_C z dx + x dy + y dz$ where c is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $y + z = 2$. Orient c counterclockwise. (10%)

(二) Let $F(x, y, z) = z\hat{j} + z\hat{k}$ represent the flow of a liquid. Find the flux of F through the surface S given by that portion of the plane $z = 6 - 3x - 2y$ in the first octant oriented upward. (10%)

(三)(i) Let A be a real symmetrical matrix. Prove eigenvectors associated with distinct eigenvalues of A are orthogonal. (10%)
(ii) Prove the eigenvalues of a unitary matrix have the absolute value 1. (10%)

(四) Show that $\int_0^{\infty} e^{-x^2} \cos(2\beta x) dx = \frac{1}{2} \sqrt{\pi} e^{-\beta^2}$ for any positive number β by means of residue theorem. (15%)

(五) Solve $K \frac{\partial^2 u}{\partial x^2} + r = \frac{\partial u}{\partial t}$ subject to (20%)
 $u(0, t) = 0, u(1, t) = u_0 \quad t > 0$
 $u(x, 0) = f(x) \quad 0 < x < 1$
 where K, r are real constants

(六)(i) Derive the Laplace's equation from rectangular coordinate to polar coordinate.
(ii) Use the Laplace equation to solve the steady-state temperature $u(r, \theta)$ in the semicircular plate shown in the following figure, assume that $u(r, \theta)$ is bounded at $r=0$. (25%)

