國立臺灣科技大學
八十八學年度碩士班招生考試試題

系所別：工程技術研究所碩士班 自動化及控制學程　組別：乙組　科目：作業研究

Show your calculations in details.

1. Consider the following linear programming problem:

$$\begin{aligned}
\text{Minimize } z = \; & 2x_1 + x_2 \\
\text{Subject to} \quad & 3x_1 + x_2 - x_3 = 3 \\
& 4x_1 + 3x_2 - x_4 = 6 \\
& x_1 + 2x_2 + x_5 = 3 \\
& x_1, x_2, x_3, x_4, x_5 \geq 0
\end{aligned}$$

Compute the entire simplex tableau associated with the follow basic solution and check its feasibility and optimality (25%).

$$\mathbf{x}_B = \begin{bmatrix} x_1 \\ x_2 \\ x_5 \end{bmatrix}, \qquad \mathbf{B}^{-1} = \begin{bmatrix} 3/5 & -1/5 & 0 \\ -4/5 & 3/5 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

2. Consider the following bounded variable linear programming problem:

$$\begin{aligned}
\text{Maximize } z = \; & x_1 - x_2 + x_3 - x_4 \\
\text{Subject to} \quad & x_1 + x_2 + x_3 + x_4 = 8 \\
& 0 \leq x_1 \leq 8 \\
& -4 \leq x_2 \leq 4 \\
& -2 \leq x_3 \leq 4 \\
& 0 \leq x_4 \leq 10
\end{aligned}$$

(a) Write the dual of the above problem (10%).

(b) Show that $x_1 = 8$, $x_2 = -4$, $x_3 = 4$ and $x_4 = 0$ is an optimal solution by using the complementary slackness conditions (15%).

3. A businessman is planning to open a car-wash shop and he must decide how much space to provide for waiting cars. It is estimated that customers will arrive according to a Poisson process with a mean rate of 1 every 4 minutes, unless the waiting area is full in which case the arriving customers will take their cars

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elsewhere. The shop can only wash one car at a time and the time that required to wash a car has an exponential distribution with a mean of 3 minutes. Compare the expected fraction of customers that will be lost because of inadequate waiting space if (a) 0 space (not including the car being washed), (b) 2 spaces, and (c) 4 spaces were provided (25%).

4. A n industrial robot has four axes of motion, controlled by four separate motors. These motions are referred as "Turn", "Lift", "Extend", and "Grip". When the robot is programmed to perform a task, a sequence of these motions will take place. For example, to pick up an object, the arm would turn to align with the object, extend to reach it, grip it, and lift. It is important to predict how the actions will occur in order to accelerate the controlling software. For example, if a turn is most likely to be followed by an extension, the code that controls the extension can be read in advance. For a sample of robot programs, representing a set of typical tasks, the following count data was derived:

		Next motion:			
		T	L	E	G
Current motion:	T	5	40	90	15
	L	50	0	20	80
	E	30	40	20	60
	G	15	90	30	15

Construct a Makov chain and show how to address the following questions.

(a) If the arm is turning, how many action steps will occur on the average before the next grip action is taken (9%)?

(b) If a grip occurs, what is the probability that another grip occurs within the next four actions (8%)?

(c) Over the long run, how does the usage of the lift motor compare to the usage of the extension motor (8%)?