國立臺灣科技大學
八十八學年度碩士班招生考試試題

系所別：工程技術研究所碩士班 自動化及控制學程　組別：乙組　科目：統計學

STATISTICS

1.(20%) Suppose that a random sample of size $n = 100$ is sampled from a normal population with unknown mean μ and variance $\sigma^2 = 40^2$. The goal is to test $H_0 : \mu = 50$ against $H_1 : \mu > 50$ at 2.5% significance level. Compute the power when $\mu = 62$. If the sample mean $\bar{X} = 54$ is observed, what is the p-value and your conclusion? ($\mathrm{P}\{Z > 2\} \approx 0.025, \mathrm{P}\{Z > 1\} \approx 0.16$, where Z stands for the standard normal random variable.)

2.(10%) When estimating $\mu_X - \mu_Y$, we collect two samples of equal size $\{X_1, X_2, \ldots, X_n\}$ and $\{Y_1, Y_2, \ldots, Y_n\}$ and use $\bar{X} - \bar{Y}$ to estimate $\mu_X - \mu_Y$. Two options are possible: (1) *blocking* — making X_i and Y_i positively correlated for every i, and (2) independent samples. State the advantage and disadvantage of using *blocking*.

3.(50%)
(a) State the *importance* of the *Central Limit Theorem*. (No need to give its definition.)
(b) A random sample $\{X_1, X_2, \ldots, X_{100}\}$ is collected to estimate $\mathrm{E}[X_i]$. Suppose that the sample mean $\bar{X} = \sum_{i=1}^{100} X_i/100 = 25.67899$ and sample variance $S^2 = \sum_{i=1}^{100}(X_i - \bar{X})^2/(100-1) = 0.0001$ are observed. Identify the significant digits of $\bar{X}$; that is, identify the digits in $\bar{X}$ that are very likely to be the same as those in $\mathrm{E}[X_i]$.
(c) Why do we need to learn *probability* before *statistics*?
(d) When estimating parameter θ by an unbiased estimator $\hat{\theta}$, why do we need to assess the variability of $\hat{\theta}$?
(e) Suppose that sample mean $\bar{X} = 45.2$ of a sample drawn from a population with mean μ is observed. Can you give the conclusion when testing $H_0 : \mu = 1000$ against $H_1 : \mu < 1000$? Why or why not?

4.(10%) Urn A contains 1 black ball and 1 white ball. Urn B contains 2 black balls and 1 white ball. Randomly select one ball from urn A and place unseen in urn B. Then randomly select one ball from urn B.
(a) What is the probability that the balls selected from urns A and B are both black?
(b) Given that the ball selected from urn B is black, what is the probability that the ball selected from urn A and placed unseen in urn B is black?

5.(10%) A *straight* is five cards having consecutive denominations *but not all in the same suit*–for example, a 4 of diamonds, 5 of hearts, 6 of hearts, 7 of clubs, and 8 of diamonds. An ace may be counted "high" or "low," which means that (10, jack, queen, king, ace) is a straight and so is (ace, 2, 3, 4, 5). (If five consecutive cards are all in the same suit, the hand is called a *straight flush* and is considered a fundamentally different type of hand in the sense that a straight flush "beats" a straight.) Give the expression for the probability of getting a *straight*.

Hand	Probability
One pair	0.42
Two pairs	0.048
Three-of-a-kind	0.021
Straight	0.0039
Flush	0.0020
Full house	0.0014
Four-of-a-kind	0.00024
Straight flush	0.000014
Royal flush	0.0000015