

國立臺灣科技大學
八十八學年度碩士班招生考試試題

系所別：資訊管理系碩士班

組別：丙組

科目：作業研究

Operations Research

1. Let

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix}.$$

- (a) Find orthonormal vectors $\vec{q}_1, \vec{q}_2, \vec{q}_3$ such that $\vec{q}_1, \vec{q}_2$ span the column space of A . (5%)
 - (b) Solve $A\vec{x} = \vec{b} = [1 \ 2 \ 7]^T$ for $\vec{x}$ by least squares. (5%)
 - (c) Express the projection of $\vec{b}$ onto the column space of A , $\vec{p}$, as a linear combination of $\vec{q}_1$ and $\vec{q}_2$. (5%)
2. Show that A and $A^T A$ have the same nullspace. (10%)
 3. Find the shortest distance between the point (x_0, y_0, z_0) and the plane $\{(x, y, z) | ax + by + cz = 0\}$. (10%)
 4. Suppose that a random sample of size $n = 100$ is sampled from a normal population with unknown mean μ and variance $\sigma^2 = 40^2$. The goal is to test $H_0 : \mu = 50$ against $H_1 : \mu > 50$ at 2.5% significance level. Compute the power when $\mu = 62$. If the sample mean $\bar{X} = 54$ is observed, what is the p -value and your conclusion? ($P\{Z > 2\} \approx 0.025, P\{Z > 1\} \approx 0.16$, where Z stands for the standard normal random variable.) (10%)
 5. How many ways are there to place $r (\geq n)$ indistinguishable balls into n distinguishable boxes such that no box is empty? (5%)
 6. A processor for refining sugar yields up to 1 ton of pure sugar per day, but the actual amount produced Y , is a random variable because of machine breakdowns and other slowdowns. Suppose Y has a density function given by

$$f(y) = \begin{cases} 2y, & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

The company is paid at the rate of \$350 per ton for the refined sugar, but it also has a fixed overhead cost of \$120 per day. Thus the daily profit, in hundreds of dollars, is $U = 3.5Y - 1.2$. Find the probability density function for U . (15%)
 7. Suppose that player A has \$2 and with each play of the game wins \$1 with probability $p > 0$ or loses \$1 with probability $1 - p$. The game ends when player A either accumulates \$4 or goes broke. Formulate this problem as a Markov chain. Calculate the probability that A will win \$4 and the probability that player A will go broke if $p = 0.45$. (10%)

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8. Consider the following linear program:

$$\text{Maximize } 15x_1 + 30x_2 + 20x_3$$

s.t.

$$\begin{aligned} x_1 &+ x_3 &\leq 4 \\ 0.5x_1 + 2x_2 + x_3 &\leq 3 \\ x_1 + x_2 + 2x_3 &\leq 6 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

- (a) Write down the dual problem.(5%)
 (b) Solve both the primal and the dual problems optimally.(5%)
 (c) What is the dual prices for the primal constraints?(5%)
9. the coach of a school swim team needs to assign swimmers to a 200-yard medley relay team to send to the Junior Olympics. Since most of his best swimmers are very fast in more than one strokes, it is not clear which swimmer should be assigned to each of the four strokes. The five fastest swimmers and the best times (in seconds) they have achieved in each of the strokes (for 50 yards) are as follows:

Stroke	Carl	Chris	David	Tony	Ken
Backstroke	38	33	34	37	35
Breaststroke	43	33	42	34	42
Butterfly	33	29	39	30	34
Freestyle	29	26	30	29	31

Determine how to assign four swimmers to the four different strokes to minimize the sum of the corresponding best times.(10%)