

**國立臺灣科技大學**  
八十八學年度碩士班招生考試試題

系所別：化學工程系碩士班

組別：

科目：工程數學

共一百分。可不按順序作答，但請標明題號，詳列過程，否則酌予扣分。

1. (10 %) Solve the differential equation :  $(1+x^2)y' - 2xy = 1+x^2$ ,  $y(0) = 1$
2. (15 %) Given that  $y_1 = x$  is a solution of the homogeneous differential equation :  
 $(x^2 - 1)y'' - 2xy' + 2y = 0$ . Solve the non-homogeneous differential equation :  
 $(x^2 - 1)y'' - 2xy' + 2y = x^2 - 1$   
 (Hint: find  $y_2$  of the homogeneous differential equation by the method of reducing order, then find the particular solution for the non-homogeneous differential equation by the method of variation of parameters)

3. (10 %) Express  $\exp(A)$  as a  $2 \times 2$  matrix if

$$A = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$$

(Hint:  $\exp(A) = I + A + A^2/2! + \dots + A^n/n! + \dots$ , then calculate  $A^n$ )

4. (5 %) Prove the following vector identity :  $\nabla \bullet (\mathbf{u} \times \mathbf{v}) = \mathbf{v} \bullet \nabla \times \mathbf{u} - \mathbf{u} \bullet \nabla \times \mathbf{v}$   
 Bold-faced characters represent vector,  $\times$  and  $\bullet$  represent cross product and inner product, respectively.
5. (10%) By using surface integral, calculate the surface area of surface  $S$  whose parametric representation is  $\mathbf{r}(u, v) = \mathbf{i}(a + b \cos v) \cos u + \mathbf{j}(a + b \cos v) \sin u + \mathbf{k}(b \sin v)$ ,  
 $0 \leq u \leq 2\pi$ ,  $0 \leq v \leq 2\pi$
6. (15 %) Solve the differential equation :  
 $y'' + 4y = f(t)$ ;  $y(0) = 0$ ,  $y'(0) = 1$ ,  $f(t) = 0$  if  $t < 3$  and  $f(t) = 1$  if  $t \geq 3$ .
7. (15 %) Find the solution of the differential equation :  $2x y'' + (2x+1) y' + 2y = 0$   
 near  $x = 0$ .
8. (20 %) Solve the boundary value problem :

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \quad (0 < x < 2, t > 0)$$

$$y(0, t) = y(2, t) = 0 \quad (t > 0)$$

$$y(x, 0) = 0 \quad (0 < x < 2)$$

$$\frac{\partial y(x, 0)}{\partial t} = g(x) \quad (0 < x < 2)$$