

國立臺灣科技大學
八十八學年度碩士班招生考試試題

系所別：電機工程系碩士班

組別：甲組

科目：工程數學

共十題，滿分 100 分。

1. Derive the general solution for the following differential equation

$$xy' = 2\frac{y^2}{x} + y \quad (10\%)$$

2. Apply the Laplace transform to solve the following initial value problem

$$y'' + 2y' + 2y = \delta(t-2); \quad y(0) = y'(0) = 0 \quad (10\%)$$

3. (a) Prove Cauchy-Schwarz inequality
- $|\vec{F} \cdot \vec{G}| \leq \|\vec{F}\| \|\vec{G}\|$
- , where
- \vec{F}
- and
- \vec{G}
- are vectors. (5%)

- (b) Use Cauchy-Schwarz inequality to verify
- $\|\vec{F} + \vec{G}\| \leq \|\vec{F}\| + \|\vec{G}\|$
- (5%)

4. (a) Derive the Fourier transform
- $F\{e^{-a|t|}\} = \frac{2a}{a^2 + \omega^2}$
- for any positive "a". (5%)

- (b) By using the Fourier transform shown in (a), find the Fourier transform of the function given below:

$$f(t) = 10e^{-3|t+1|} \quad (5\%)$$

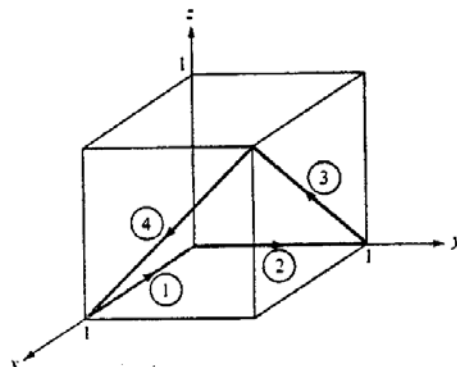
5. Determine the eigenvalues of the matrix given below. Also, find an eigenvector for each eigenvalue. (10%)

$$\begin{pmatrix} 3 & 0 & 0 \\ 1 & -2 & -8 \\ 0 & -5 & 1 \end{pmatrix}$$

6. Calculate the determinant of the matrix. (10%)

$$\begin{pmatrix} 1 & 3 & 4 & 2 \\ 0 & -1 & 3 & 4 \\ 2 & 1 & 9 & 6 \\ 3 & 2 & 4 & 8 \end{pmatrix}$$

7. Given that
- $\vec{F} = x^2 \hat{a}_x - xz \hat{a}_y - y^2 \hat{a}_z$
- , calculate the circulation of
- \vec{F}
- around the (closed) path shown in the following figure. (10%)



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8. Give the solutions of the following equations. (10%)

$$X_1 + 2X_2 + X_3 - X_4 = 2$$

$$X_1 + 4X_2 + 5X_3 - 3X_4 = -2$$

$$-2X_1 - X_2 + 8X_3 - 2X_4 = -10$$

$$3X_1 + 7X_2 + 5X_3 - 8X_4 = 6$$

9. Calculate $\oint \frac{dz}{z(z-2)^2(z-4)}$, where c is the counterclockwise curve of $|z|=3$ in the complex plane. (10%)

10. A curve is represented by $1 \leq |z| \leq 2$, if a conformal mapping is given by

$$w = \frac{1}{z - (1+i)}$$

draw the resulted curve. (10%)