

國立臺灣科技大學
八十八學年度碩士班招生考試試題

系所別：機械工程系研究所

組別：甲、乙、丙、丁、戊組

科目：工程數學

1. Find the general solution of the system of differential equations, where double dots denote d^2/dt^2 . (20%)

$$\ddot{x} = 2x + 4y$$

$$\ddot{y} = x - y$$

2. Given (20%)

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

Determine matrices Q and D such that $Q^{-1}AQ = D$ is diagonal.

3. Evaluate the line integral which are taken around the given contour C in the clockwise sense as viewed from the origin. (20%)

$$\int_C (\sin z dx - \cos x dy + \sin y dz),$$

C : the boundary of the rectangle

$$0 \leq x \leq \pi, \quad 0 \leq y \leq 1, \quad z=3$$

4. Apply the residue theorem to evaluate (20%)

$$I = \int_0^{2\pi} \frac{d\theta}{2 - \sin \theta}.$$

5. Solve the vibrating-string problem (20%)

$$c^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2} \quad (0 < x < l, \quad 0 < t < \infty)$$

$$y(0, t) = y(l, t) = 0, \quad y(x, 0) = F(x),$$

$$\frac{\partial y}{\partial t}(x, 0) = G(x),$$

by separation of variables, leaving expansion coefficients in integral form.