

國立臺灣科技大學
八十八學年度碩士班招生考試試題

系所別：電子工程系碩士班

組別：甲組

科目：離散數學與資料結構

1. Four people, a, b, c, and d, were in a race. Right before the race started, Dave said that he thought a would be the second and d would be the fourth, John thought a would be the first and b would be the second, and Paul thought c would be the second and d would be the fourth. If each of them had only one estimate right

a. Write all the estimates in logical propositions (Use the propositions: A= "a is the first", B= "b is the second", C= "c is the third", D= "d is the fourth", P= "c is the second", Q= "a is the second".) (5%)

b. Use the logical operators (formal proof) to find the rank of a, b, c, and d. (10%)

2. Find a recurrence relation for the number of ways to fully parenthesize the expression

$$w_1 + w_2 + w_3 + \dots + w_n$$

so that only two terms are added at a time; that is, parentheses are used to indicate the order in which the additions are performed. For example, the expression $((w_1 + w_2) + w_3)$ is fully parenthesized, but $(w_1 + w_2) + w_3$ is not. (10%)

3. Let $Z(4) = \{0, 1, 2, 3\}$. Define the binary operator $\odot$ on $Z(4)$ by

$$x \odot y = xy \pmod{4} \quad \text{for } x, y \text{ in } Z(4).$$

a. Give the complete table of operator $\odot$ for $Z(4)$. (5%)

b. Determine and justify whether $(Z(4), \odot)$ is a semigroup, a monoid, or a group. (5%)

4. Let G be a planar Hamiltonian simple graph with n vertices, and let C be a Hamiltonian cycle in G . Then with respect to C , prove $\sum (k-2)(r_k - s_k) = 0$. Here r_k is the number of faces inside C whose boundary contain exactly k edges, and s_k is the number of faces outside C whose boundary contains exactly k edges. (15%)

5. Let M be the adjacency matrix of the simple graph G ,

a. Show that the number of triangles in G is $\frac{1}{6} \|M^3\|$. Here $\|M^3\|$ denotes the sum of the elements of the main diagonal of M^3 . (5%)

b. Use the above formula, find the number of triangles in the following graph:



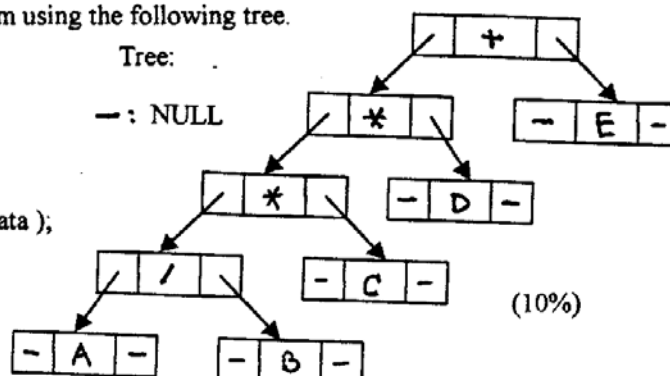
(5%)

6. Write the trace of the following program using the following tree.

Program:

```
void xx (pointer ptr)
{ if (ptr) { xx (ptr->left);
            xx (ptr->right);
            printf ("%d", ptr->data);
          }
}
```

Tree:



(10%)

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7. The following is a quicksort program.

```
void quicksort ( int i, int j )
/* sorts the records stored in array data between indexed by i and j in descending order. */
{ int p;
  if (i < j)
  { p = partition ( i, j );
    quicksort ( i, p-1);
    quicksort ( p+1, j );
  }
}
```

Write the algorithm of the inside function partition(i, j).

(15%)

8. Describe the functions(and the parameters) of the following three parts:

i. void add (pointer *ptr, pointer node)

```
{ ...
  node -> link = *ptr;
  *ptr = node;
}
```

(5%)

ii. void add (pointer *head, pointer *ptr, pointer node).

```
{ ...
  node -> link = NULL;
  if ( *head ) (ptr) -> link = node;
  else *head = node;
  *ptr = node;
}
```

(5%)

iii. void add (pointer *ptr, pointer node)

```
{ ...
  if (IS_EMPTY (*ptr))
  { *ptr = node;
    node -> link = node;
  }
  else { node -> link = (*ptr) -> link;
        (*ptr) -> link = node;
      }
}
```

(5%)