國立臺灣科技大學
八十八學年度碩士班招生考試試題

系所別：電子工程系碩士班　　　組別：丙組　　　科目：工程數學

1. a) $f(t) = e^{-\alpha t^2}$, find the Fourier transform of f(t) given that $\int_{-\infty}^{\infty} e^{-\alpha t^2} dt = \sqrt{\pi/\alpha}$ (10%)

b) Evaluate $\int_{-\infty}^{\infty} \delta(at-b)x(t)dt$ (5%)

c) Is that $\delta(t)$ an odd or even function ? Prove it (5%)

2. a) Show that if $M + Ny' = 0$ has a solution, then it has an integrating factor.(5%)
b) Solve $(x + y)dx + x \ln x\, dy = 0$ (10%).

3. The Laplace transform of x(t) is X(s) prove that $\lim_{t \to \infty} x(t) = \lim_{s \to 0} [sX(s)]$ (10%)

4. Convolution theorem states that the relationship among h(t), x(t), and y(t) is $y(t) = x(t)*h(t)$, where h(t), x(t), y(t), and * denote the system impulse response, input, output and convolution operation, respectively. If $x(t)=1-\cos(at)$, $h(t)= e^{bt}$, find $y(\infty)$. (10%)

5. If $A = \begin{bmatrix} 3 & -1 & 1 & 1 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 1 & 2 & 6 & 1 \\ 1 & 1 & -1 & 6 & 7 & 3 \\ 0 & 0 & 2 & 1 & 3 & 9 \\ 0 & 0 & 0 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$, find the eigenvalues of A and B.
(Hint : decomposeing the matrices into small size submatrices) (10%)

6. $A = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 0 \end{bmatrix}$, changing A to an orthogonal matrix (10%)

7. If $\vec{v} = x^2\vec{i} + (xy)\vec{j}$
a) Does it exist $\phi(x, y)$, such that $\nabla\phi(x, y) = \vec{v}$? If yes, find $\phi(x, y)$ (5%).
b) Find $\int_c \vec{v} \cdot \vec{dr}$, where c is $x^2 + y^2 = b^2$. Is this independent of path? (5%)

8. Evaluate $\int_0^{2\pi} \frac{\cos 2\theta d\theta}{1 - 2p\cos\theta + p^2}$ where $-1 < p < 1$. (15%)