國立臺灣科技大學
八十八學年度碩士班招生考試試題

系所別：資訊工程研究所碩士班　　組別：　　科目：離散數學

1 10% Give a type-2 grammar that specifies both case (a) and case (b) of the following languages:

(a) 4% Every sentence in the language is a string of equal numbers of a's and b's.

(b) 4% Every sentence in the language is a string of a's and b's with the number of a's being a multiple of 3.

(c) 2% Can you use type-3 grammar to specify both of the above languages?

2 10% Prove that if $T = (V, E)$, $| V | = p$ is a binary tree of height h, then $h + 1 \leq p \leq 2^{h+1} - 1$. (Hint, the level of the root of T is 0 and the height of T is the largest level of T.)

3 10% There is a radar, a computer, and a gyroscope on board an airplane. The probability that the radar fails is 0.2. If the radar fails, the gyroscope will also fail, and the probability that the computer fails is 0.3. If the radar functions correctly, then the computer will also function correctly, and the probability that the gyroscope fails is 0.2.

(a) 2% Describe the sample space.

(b) 4% What is the probability that the computer or the gyroscope function correctly, and the other does not?

(c) 4% What is the probability that the radar functions correctly if one of the other two system fails?

4 10% A student writes a kill algorithm with two major steps to solve a problem with r numbers for $r \geq 2$.
Step 1: Use 2r -1 comparisons to determine the most possible two numbers that should be killed.
Step 2: Recursively, kill the remaining r-2 numbers.
Let a_r denote the number of comparisons used for killing r numbers, where a_0=a_1=0. Determine a_r.

5 10% Let $(A, +, .)$ be a ring, where $A = \{0, 1, 2, 3, 4, 5\}$. Also let
$0 + A = \{0, 1, 2, 3, 4, 5\}$,
$1 + A = \{1, 2, 3, 4, 5, 0\}$,
$2 + A = \{2, 3, 4, 5, 0, 1\}$,
$3 + A = \{3, 4, 5, 0, 1, 2\}$,
$4 + A = \{4, 5, 0, 1, 2, 3\}$,
$5 + A = \{5, 0, 1, 2, 3, 4\}$.
and
$0 . A = \{0, 0, 0, 0, 0, 0\}$,
$1 . A = \{0, 1, 2, 3, 4, 5\}$,
$2 . A = \{0, 2, 4, 0, 2, 4\}$,
$3 . A = \{0, 3, 0, 3, 0, 3\}$,
$4 . A = \{0, 4, 2, 0, 4, 2\}$,
$5 . A = \{0, 5, 4, 3, 2, 1\}$.

(a) 3% Assume $H = \{0, 2, 4\}$ is a subset of A. Is H ideal?

(b) 7% Based on H, find a homomorphic image $(B, \oplus, \odot)$ of $(A, +, .)$.

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6. 5% (a) Consider the proposition $p(x, y): y - x = y + x^2$ where the universe consists of all integers. Determine the truth value of the following statement and explain why. $\forall y \exists x\, p(x, y)$

5% (b) Let p(x) and q(x) be propositions. Determine the truth value of the following statement and explain why. $[\exists x\, p(x)] \wedge [\exists x\, q(x)] \Leftrightarrow \exists x\, [p(x) \wedge q(x)]$

7. 10% Let f be a function from X to Y. Prove that f is one-to-one if and only if $f(A \cap B) = f(A) \cap f(B)$ for all subsets A and B of X.

8. Let R_1 and R_2 be equivalence relations on the set S. Prove or disprove that the following combinations of R_1 and R_2 are equivalence relations.

3% (a) $R_1 \cup R_2$

3% (b) $R_1 \oplus R_2$

4% (c) $R_1 \circ R_2$

9. A homomorphism from the group (G, o) to the group (H, *) is a function $\varphi : G \to H$ satisfying $\varphi(g \circ h) = \varphi(g) * \varphi(h)$ for all $g, h \in G$.

5% (a) Determine whether the function $\varphi(n) = n^2$ is a homomorphism from (Z, +) to (Z, +) or not, and explain why.

5% (b) Let $\varphi : G \to H$ be a homomorphism from group (G, o) onto group (H, *), and e denote the identity in G. Show that $\varphi(e)$ is the identity of H.

10. 10% Use mathematical induction to show that

$$\sum \frac{1}{n_1 \cdot n_2 \cdots n_k} = n$$

where the sum is taken over all nonempty subsets $\{ n_1, n_2, \ldots, n_k \}$ of $\{ 1, 2, \ldots, n \}$.