國立臺灣科技大學
八十九學年度碩士班招生考試試題

系所組別： 工程技術研究所自動化及控制學程乙組
科　目： 作業研究

1. A company has agreed to build at least one bridge across a raging river in the Taipei basin. Three sites have been chosen, but for each site there is a positive probability that the company will fail in its attempt to build a bridge at that site. These probabilities of failure for sites 1, 2 and 3 are 0.6, 0.8, 0.7, respectively. However, the probability of failure can be reduced by assigning additional teams of workers to the site. Three additional teams are available, and the following table gives the probability that a site will fail in its attempt when 0, 1, 2, or 3 additional teams are assigned to the site:

		Probability of faiure		
		Site 1	Site 2	Site 3
Number of additional teams	0	0.60	0.80	0.70
	1	0.40	0.40	0.50
	2	0.25	0.30	0.30
	3	0.20	0.20	0.20

The company wants to minimize the probability that it will fail to build a bridge across the river, so is wants to minimize the product of probabilities that each site will fail. Use dynamic programming technique to find an optimal solution (25%).

2. Carco uses robots to manufacture cars. The following demands for cars must be met (not necessarily on time, but all demands must be met by end of quarter 4): 600 units for quarter 1, 800 units for quarter 2, 500 units for quarter 3, and 400 units for quarter 4. At the beginning of quarter 1, Carco has two robots. Robots can be purchased at the beginning of each quarter. At most two robots can be purchased during any quarter. Each robot can build up to 200 cars per quarter. It costs $5,000 to purchase a robot. Each quarter, a robot incurs $500 in maintenance costs (even if it is not used to build any cars). Robots can be sold at the beginning of each quarter for $3,000. At the end of each quarter, a holding cost of $200 per car is incurred. If any demand is backlogged, a cost of $300 per car is incurred for each quarter the demand is backlogged. At the end of quarter 4, Carco must have at least two robots. Formulate a linear programming problem to minimize the total cost incurred in meeting the next four quarters' demand for cars for Carco (25%).

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3. A manufacturing firm produces four products. Each product requires material and machine processing. The linear programming model formulated to determine the number of product 1 (x_1), product 2 (x_2), product 3 (x_3), and product 4 (x_4) to produce is as follows:

$$\text{Maximize } z = 2x_1 + 8x_2 + 10x_3 + 6x_4 \text{ (profit, \$)}$$

$$\text{Subject to } 2x_1 + x_2 + 4x_3 + 2x_4 \le 200 \text{ (material, lb)}$$

$$x_1 + 2x_2 + 2x_3 + x_4 \le 160 \text{ (machine processing, hours)}$$

$$x_1, x_2, x_3, x_4 \ge 0$$

Let x_5 and x_6 represent the slack variables for the material and the machine processing constraints, respectively. The optimal simplex tableau is:

Basis	x_1	x_2	x_3	x_4	x_5	x_6	RHS
x_4	1	0	2	1	2/3	− 1/3	80
x_2	0	1	0	0	− 1/3	2/3	40
$c_j - z_j$	− 4	0	− 2	0	− 4/3	− 10/3	800

a. What is the marginal value of an additional pound of material? Over what range is this value valid (15%)?
b. How much would the unit profit of product 1 have to increase before it would be produced (10%)?

4. Partially completed products arrive at a workstation in a factory at a mean rate of 40 per hour and is Poisson distributed. The processing time at the workstation averages 1.2 minutes per unit and is exponentially distributed. The factory estimates that each unit of in-process inventory at the workstation cost $31 per day on the average. However, the factory can add extra workers and reduce the processing time to 0.90 minute per unit at a cost of $52 per day. Determine whether the factory should continue the present operation or add extra workers (25%).