國立臺灣科技大學

八十九學年度碩士班招生考試試題

系所組別： 工程技術研究所自動化及控制學程乙組

科　目： 統計學

STATISTICS

按題目順序作答, 否則扣分

1. What is *statistic*? Give one example. Why is *statistic* a random variable? What is the purpose of learning the *sampling distribution* of a *statistic*?(20%)

2. A pond consists of t fish. To estimate the unknown t, the following steps are taken:

   (i) Randomly catch n $(n < t)$ fish from the pond and tag each of them.

   (ii) Put n tagged fish back to the pond.

   (iii) Randomly catch m $(m < t)$ fish and X tagged fish are found.

   One principle of constructing a confidence interval for an unknown parameter is to include the parameter values that have high probability (e.g. $(1-\alpha)$) in observing the sample. Give detailed algorithm for constructing a $(1-\alpha)100\%$ confidence interval for t?(10%)

3. (a) If 9 heads is observed when tossing a coin 10 times, do you believe that the coin is a fair coin? Why or why not?(10%)

   (b) Explain why the *maximum likelihood estimator* is a reasonable estimator.(10%)

4. Let U_1 and U_2 be independent random variables with common density

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Find the mean of $\min(U_1, U_2)$.(10%)

   (b) Find the joint density of $\min(U_1, U_2)$ and $\max(U_1, U_2)$.(10%)

5. Suppose that a random sample of size $n = 100$ is sampled from a normal population with unknown mean μ and variance $\sigma^2 = 40^2$. The goal is to test $H_0 : \mu = 50$ against $H_1 : \mu < 50$ at 2.5% significance level. Compute the power when $\mu = 38$. $(\mathrm{P}\{Z > 2\} \approx 0.025, \mathrm{P}\{Z > 1\} \approx 0.16$, where Z stands for the standard normal random variable.)(10%)

6. Consider the one-way ANOVA model:

$$Y_{ij} = \mu_i + \varepsilon_{ij}, \qquad i = 1, 2, \ldots, k, \quad j = 1, 2, \ldots, n,$$

   where Y_{ij} denotes the jth observation from population i and k denotes the number of populations. The goal is to test $H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$ against H_1 : not H_0.

   (a) State the required assumptions.(10%)

   (b) Give detailed test procedure.(10%)