

國立臺灣科技大學  
八十九學年度碩士班招生考試試題

系所組別：機械工程系丙組  
科目：流體力學

**Note:** All the governing equations and mathematical relations needed are listed in page 3.

1. True or false:

每題2分，共10分

- 1) If the Bernoulli equation is valid throughout the flow field for a particular flow, then this flow must be an irrotational flow.
- 2) For Newtonian flow, the stress ( $\bar{\sigma}$ ) and rate of stress tensor ( $\bar{\epsilon}$ ) are symmetric.
- 3) In general, viscous force can be expressed as ( $div \bar{\tau}$ ) which can be reduced to  $\mu \nabla^2 \vec{V}$  for a steady, constant viscosity, incompressible, Newtonian flow.
- 4) Surface stress tensor is the summation of normal stress and tangential stress tensor.
- 5) For fluid element of an incompressible flow, the velocity can not decrease in every direction.

2. Regards the conservation of mass, please

12 %

- a) Derive the differential form of continuity equation from the integral form,
- b) Expand the above answer in part (a) for spherical coordinate system.

6 %

6 %

Note: Do not write down the answers directly.

3. Please answer the following questions regarding the momentum equation.

12%

- a) Explain the physical meaning for each term in the differential momentum equation,
- b) Expand the acceleration term of a particle for x-direction in Cartesian coordinates,
- c) Regards the acceleration of a particle, state the procedure (or steps) how to expand it in orthogonal curvilinear coordinates. Note that you are not supposed to expand it.

4. Answer the following questions briefly

每題3分，共36分

- 1) Write down the relation between vorticity, circulation, and rotation.
- 2) What is the dimension of stream function? Which fundamental law is automatically satisfied by substituting the definition of stream function?
- 3) What is the working principle for a hydraulic lift?
- 4) What kinds of flow characteristics are needed for potential function to satisfy the Laplace equation? What is the dimension of potential function?
- 5) List the physical meaning and the mathematical expression for Reynolds Transport Theorem.
- 6) Why should we need to use "Incomplete Similarity"? How does this concept work in practical experimental test?
- 7) What are the "entrance length" and "fully developed flow"?
- 8) For a fully developed flow in a pipe, we have the expression for volume flow rate as

$$Q = -(\pi R^4 / 8\mu)(\partial p / \partial x)$$

state how it can be used to measure the viscosity of fluid.

- 9) Capillary effect is responsible for the transportation of water from the root to the top of a tree, please explain this phenomena.
- 10) The dimples (小凹洞) on gulf ball play an essential role for reducing the drag; because of this effect, an undimpled ball has a shorter trajectory than a dimpled one. Please explain it.
- 11) Arrange the order for the sound speed in the air, water, and steel, also explain your answer briefly.
- 12) For a flow past a plate, besides the Blasius similarity solution, Von K'arm'an also developed the famous Momentum Integral Equation. What is the difference between them, and why Von K'arm'an study this problem again?



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每小題3分，共12分

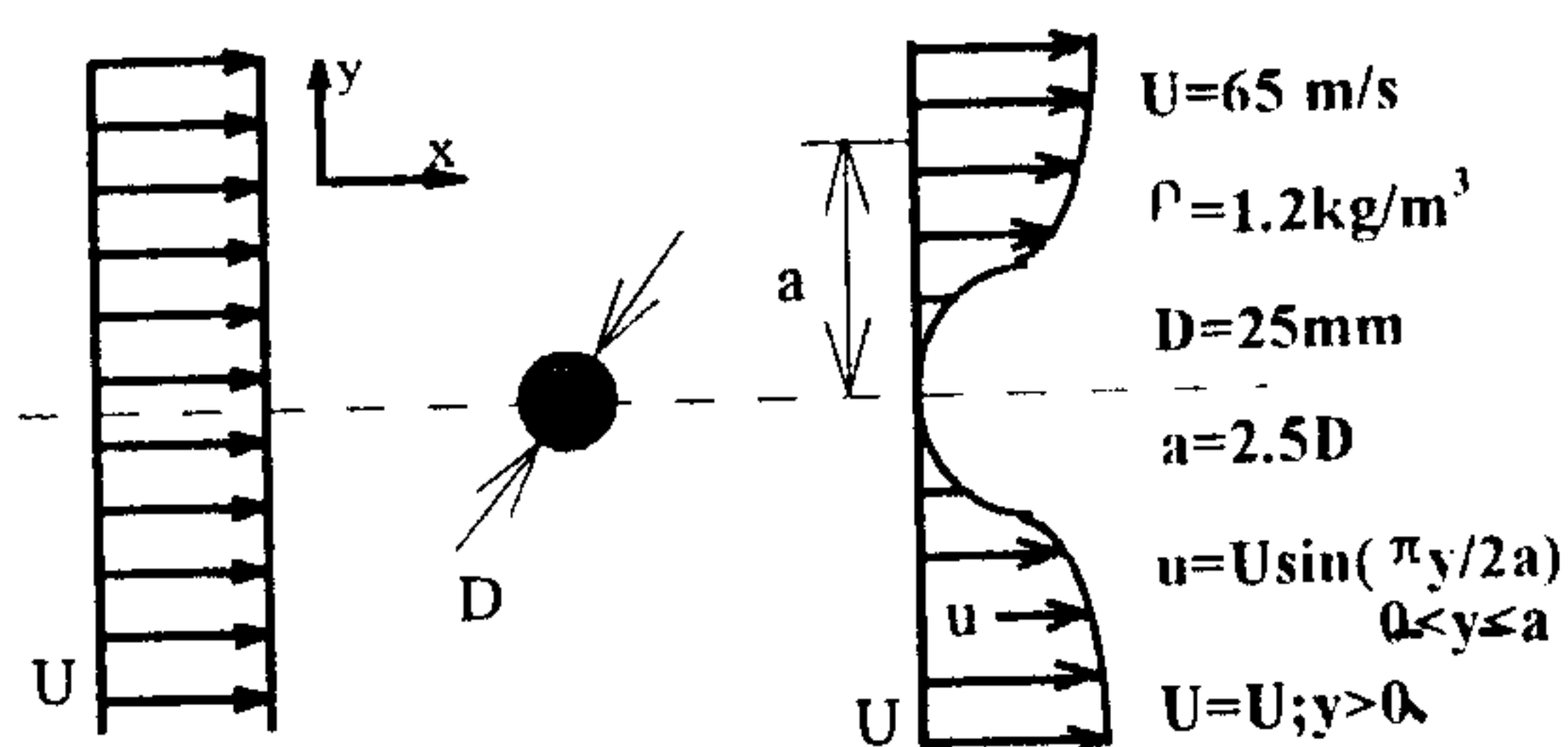
5. For a velocity field as

$$\vec{V} = -\frac{U}{r^2} \cos \theta \hat{e}_r - \frac{U}{r^2} \sin \theta \hat{e}_\theta$$

where  $U$  is a constant.

- (a) Is it an incompressible flow?  
 (b) Is it an irrotational flow?  
 (c) Determine the expression for stream function,  
 (d) Obtain the expression for the acceleration.

6. Experimental measurements are made in a low-speed air jet to determine the drag force on a body shown on the figure. Velocity measurements at two sections, where the pressure is uniform and equal, give the results shown. Evaluate the drag force on this body, per unit width. 10%



7. The power  $P$  and head  $H$  ( $L^2/t^2$ ) of fan is a function of its volume flow rate  $Q$ , air density  $\rho$ , angular speed  $\omega$  ( $1/t$ ), impeller diameter  $D$ , and fluid viscosity  $\mu$ . Please use the  $\Pi$  theorem to derive the corresponding nondimensional parameters. 8%
- Note: Please pick  $D$ ,  $\omega$ , and  $\rho$  as repeating parameters.



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### Summary of Equations

**(1) Continuity Equation:**

$$0 = \frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \vec{V} \cdot d\vec{A}$$

$$\nabla \cdot \rho \vec{V} + \frac{\partial \rho}{\partial t} = 0 \quad \text{or} \quad \frac{D\rho}{Dt} + \rho \operatorname{div} \vec{V} = 0$$

**(2) Momentum Equation:**

$$\vec{F}_S + \vec{F}_B - \int_{cv} [\vec{a}_{rf} + 2\vec{\omega} \times \vec{V}_{xyz} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \dot{\vec{\omega}} \times \vec{r}] \rho dV$$

$$= \frac{\partial}{\partial t} \int_{cv} \vec{V}_{xyz} \rho dV + \int_{cs} \vec{V}_{xyz} \rho \vec{V}_{xyz} \cdot d\vec{A}$$

$$\rho \frac{D\vec{V}}{Dt} = -\nabla p + \operatorname{div} \vec{\tau} + \rho \vec{g}$$

**(3) The First Law of Thermodynamics:**

$$\dot{Q} - \dot{W}_s - \dot{W}_{\text{shear}} - \dot{W}_{\text{other}} = \frac{\partial}{\partial t} \int_{cv} e \rho dV + \int_{cs} \left( u + pv + \frac{V^2}{2} + gz \right) \rho \vec{V} \cdot d\vec{A}$$

**(4) Mathematics relation:**

**(a) Gradient Theorem**

$$\iiint_V \nabla \phi d\tau = \iint_S \phi \hat{n} dS$$

**(b) Divergence Theorem**

$$\iiint_V \operatorname{div} \vec{A} d\tau = \iint_S \vec{A} \cdot \hat{n} dS$$

**(c) Curl Theorem**

$$\iiint_V \operatorname{curl} \vec{A} d\tau = \iint_S \hat{n} \times \vec{A} dS$$

**(d) Gradient**

$$\nabla \phi = \frac{\hat{e}_1}{h_1} \frac{\partial \phi}{\partial q^1} + \frac{\hat{e}_2}{h_2} \frac{\partial \phi}{\partial q^2} + \frac{\hat{e}_3}{h_3} \frac{\partial \phi}{\partial q^3}$$

**(e)** 
$$\operatorname{div} \vec{A} = \frac{1}{h_1 h_2 h_3} \left[ \frac{\partial}{\partial q^1} (h_2 h_3 \hat{A}_1) + \frac{\partial}{\partial q^2} (h_1 h_3 \hat{A}_2) + \frac{\partial}{\partial q^3} (h_1 h_2 \hat{A}_3) \right]$$

**(f)** 
$$\operatorname{curl} \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{e}_1 & h_2 \hat{e}_2 & h_3 \hat{e}_3 \\ \frac{\partial}{\partial q^1} & \frac{\partial}{\partial q^2} & \frac{\partial}{\partial q^3} \\ h_1 \hat{A}_1 & h_2 \hat{A}_2 & h_3 \hat{A}_3 \end{vmatrix}$$

**(g)** 
$$\vec{V} \cdot \nabla \vec{V} = \nabla \left( \frac{V^2}{2} \right) - \vec{V} \times \operatorname{curl} \vec{V}$$

**(h)** 
$$\operatorname{div} \vec{\tau} = \mu \nabla^2 \vec{V} + (\lambda + \mu) \operatorname{grad}(\operatorname{div} \vec{V}) + 2 \operatorname{grad} \mu \cdot \vec{e} + (\operatorname{div} \vec{V}) \operatorname{grad} \lambda$$

**(i)** 
$$\nabla^2 \vec{V} = \operatorname{grad}(\operatorname{div} \vec{V}) - \operatorname{curl} \operatorname{curl} \vec{V}$$

