國立臺灣科技大學

八十九學年度碩士班招生考試試題

系所組別： 纖維及高分子工程系丙組

科　目： 工程數學

1. For what value of k is the function $(y + x)^k$ an integrating factor for the differential equation

$$[(y + x)\ln(y + x) + y]\frac{dy}{dx} + y = 0$$

and find the general solution. (17%)

2. Solve the initial value problem

$$y'' + 8y' + 16y = f(t) \qquad (t \geq 0) \qquad y(0) = y'(0) = 0$$

where

$$f(t) = \begin{cases} 1 - t & (0 < t < 1) \\ 0 & (t > 1) \end{cases}$$

(18%)

3 . **Solve the following problems :**

(a) Find the eigenvalues and eigenvectors of the matrix.

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

(12%)

(b) Using Green's theorem, evaluate the line integral $\oint_C \mathbf{F}(\mathbf{r}) \circ d\mathbf{r}$ counterclockwise around C, where $\mathbf{F} = y\mathbf{i} - x\mathbf{j}$, C the circle $x^2 + y^2 = 1/4$

(12%)

(c) Represent the following function $f(x)$ by a Fourier cosine series and graph the corresponding periodic extension of $f(x)$. $f(x) = x^2$ $(0 < x < L)$ (11%)

4 解下列微分方程式

(1) $y(t) = t + \int_0^t y(\tau)(t - \tau)d\tau$ (15%)

(2) $y'' - 2y' + y = 1 + e^x$ (15%)