國立臺灣科技大學

八十九學年度碩士班招生考試試題

系所組別：電子工程系甲組、電機工程系丙組、資訊工程研究所

科　目：離散數學

1. 11% Find all the inverses of 3 *modulo* 7.

2. 11% Show that there is no procedure that solves the halting problem.

3. 11% Find the sum-of-products expansion for the function $F(x, y, z) = (x + y)\bar{z}$.

4. 13% Determine the truth value of each of the following statements. Please write "true" or "false" on your answering sheet.

   (a) (3%) Assume that the sentence "If you did not study hard, then you will not pass the examination." is true. Then the sentence "If you studied hard, then you will pass the examination." is also true.

   (b) (3%) Let p, q, r be propositions. The compound propositions $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are logically equivalent.

   (c) (3%) The negation of the sentence "Everybody loves somebody." is "Somebody loves nobody."

   (d) (4%) The following argument is valid:
   If I ate my hat, then 2=3.
   I ate my hat.
   .....
   Therefore, 2=3.

5. 10% Let $S_{n,k}$ denote the number of ways to partition an n-element set into exactly k nonempty subsets. The order of the subsets is not taken into account. Find the value of the following numbers and show your answer.

   (a) (3%) $S_{3,2}$

   (b) (3%) $S_{4,2}$

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(c) (4%) $S_{n,2}$ for $n \geq 2$

6. 10% There are 10 copies of one book and one copy each of 10 other books.

(a) (5%) In how many ways can these books be arranged on a shelf?

(b) (5%) In how many ways can we select 10 books from these books?

7. 10% An ordered n-tuple $(d_1, d_2, \ldots, d_n)$ of nonnegative integers is said to be graphical if there exists a linear graph with no self-loops that has n vertices with the degrees of the vertices being $d_1, d_2, \ldots, d_n$.

(a) 5% Is (4, 3, 2, 2, 1) graphical? Why?

(b) 5% Is (3, 3, 3, 1) graphical? Why?

8. 12%

(a) 6% Let R be a binary relation on the set of all positive integers such that

$$R = \{(a, b) | a - b \textit{ is an odd positive integer}\}$$

Is R reflexive? Symmetric? Antisymmetric? Transitive? An equivalence relation? A partial ordering relation?

(b) 6% Repeat part (a) if

$$R = \{(a, b) | a = b^2\}.$$

9. 12% By properly coloring a graph we mean to paint the vertices of the graph with one or more distinct colors in such a way that no two adjacent vertices are painted with the same color.

(a) 4% What is the minimum number of colors that is needed to properly color the graph in Fig. 1.(a)?

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(b) 4% What is the minimum number of colors that is needed to properly color the graph in Fig. 1.(b)?

(c) 4% What is the minimum number of colors that is needed to properly color the graph in Fig. 1.(c)?

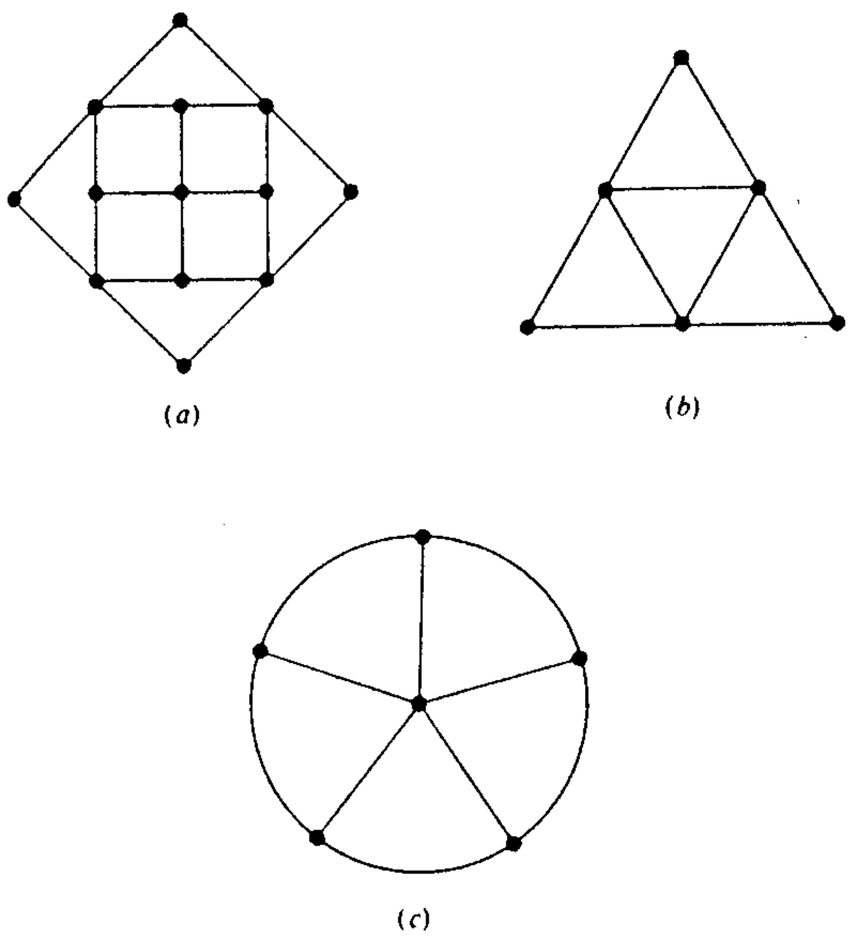

Figure 1