國立臺灣科技大學
八十九學年度碩士班招生考試試題

系所組別： 電子工程系乙一組
科　　目： 通信系統

(總分共 100 分)

1. Let us consider a continuous-time signal

$$x(t) = A\cos 2\pi f_c t \cdot (u(t+\frac{T}{2}) - u(t-\frac{T}{2}))$$

where $u(t)$ is the unit step function, defined by

$$u(t) = \begin{cases} 1, & t \geq 0\,, \\ 0, & t < 0\,. \end{cases}$$

(a). (5%) Find the Fourier transform of $x(t)$.

(b). (5%) Find the Fourier transform of $x(t)\cos 2\pi f_c t$.

2. Consider an NRZ (non-return-to-zero) wavefrom

$$x(t) = \sum_{n=-\infty}^{\infty} d_n p(t-nT)$$

where d_n is a discrete random variable with probability distribution

$$\mathrm{Prob}(d_n = 1) = \mathrm{Prob}(d_n = -1) = \frac{1}{2}$$

and

$$p(t) = A(u(t) - u(t-T))$$

with $u(\cdot)$ being the unit step function (as defined in Problem **1**).

(a). (5%) Find the autocorrelation function of $x(t)$.

(b). (5%) Find the power spectral density of $x(t)$.

3. Let $s(t)$ be a bandpass signal, centered at frequency f_c. The complex envelope (also called baseband equivalent representation) is defined to be $\tilde{s}(t)$ such that $s(t) = \mathrm{Re}[\tilde{s}(t)e^{j2\pi f_c t}]$, where $\mathrm{Re}[\cdot]$ denotes taking the real part of a complex number. Now, let us consider a QPSK (quadrature phase shift keying) signal

$$x(t) = x_c(t)\cos 2\pi f_c t - x_s(t)\sin 2\pi f_c t\,,$$

where $x_c(t)$ and $x_s(t)$ are NRZ waveforms. Let the spectrums (i.e. Fourier transforms) of $x(t)$ and $\tilde{x}(t)$ (i.e. the complex envelope of $x(t)$) be denoted by $X(f)$ and $\tilde{X}(f)$, respectively. The positive-frequency part of $X(f)$ is denoted by $X_0(f)$. That is,

$$X_0(f) = X(f)u(f)$$

where $u(\cdot)$ is the unit step function (as defined in Problem **1**).

(a). (5%) Write down the expression for $\tilde{x}(t)$ in terms of the relevant quantities that specify $x(t)$.

(b). (5%) Show that $\tilde{X}(f) = 2X_0(f+f_c)$.

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4. This problem is concerned with the sampling of continous-time signals. Under consideration are two signals $x(t)$ and $y(t)$, which are bandlimited to 20 KHz and 32 KHz, respectively.

(a). (5%) Is sampling a linear operation ?

(b). (5%) Find the Nyquist sampling rate for $x(2t) + y(t)$.

(c). (5%) Find the Nyquist sampling rate for $x(t)y(t)$.

5. Consider two versions of amplitude modulations :

$$x_{AM1}(t) = A_c(1 + \mu x(t)) \cos 2\pi f_c t \ .$$

$$x_{AM2}(t) = A_c x(t) \cos 2\pi f_c t \ .$$

where μ is the modulation index chosen to make $|\mu x(t)| < 1$ and $x(t)$ is the baseband message signal. Assume that the spectrum of $x(t)$ is given by

$$X(f) = A \cos(\frac{\pi}{2}\frac{f}{W}) \cdot (u(f+W) - u(f-W))$$

where $u(\cdot)$ is the unit step function (as defined in Problem 1).

(a). (5%) Plot the spectrum of $x_{AM1}(t)$.

(b). (5%) Plot the spectrum of $x_{AM2}(t)$.

(c). (5%) Compare the advantages and disadvantages of adopting $x_{AM1}(t)$ and $x_{AM2}(t)$ for the transmission of the message signal.

(d). (5%) One of the two listed amplitude-modulation signals can be easily demodulated with an envelope detector. Please construct a simple envelope detector circuit with a resistor and a capacitor. You must clearly specify the input end and the output end of the circuit.

6. A frequency-modulation signal is given by

$$x_c(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t x(\tau)\, d\tau) \ ,$$

where $x(t)$ is the baseband message signal. Let us consider the case that $x(t)$ is a single tone. That is,

$$x(t) = A_m \cos(2\pi f_m t) \ .$$

(a). (5%) What is the maximum frequency deviation of $x_c(t)$?

(b). (5%) Assume that the maximum frequency deviation is close, but not equal, to zero. Find the frequencies where the nonzero components of the line spectrum of $x_c(t)$ are located. In other words, when $x_c(t)$ is represented as a sum of some sinusoidal functions, what are the frequencies of these sinusoidals ?

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7. Consider a phase-modulation signal

$$x_c(t) = A_c \cos(2\pi f_c t + k_p x(t))$$

where $x(t)$ is the baseband message signal and k_p is the modulation index. Let $S_{xx}(f)$ denote the power spectrum of $x(t)$. Assume that the values of $S_{xx}(f)$ are very close to zero (and therefore negligible) for $|f| > W$.

(a). (5%) If $|k_p x(t)| \ll 1$ is assumed, what is the bandwidth of $x_c(t)$? Your answer should be expressed in terms of W.

(b). (5%) Does the phase modulation belong to the class of linear continuous-wave modulation ?

8. In BPSK (binary phase shift keying) modulation, logical 0 and logical 1 are, respectively, mapped into waveforms

$$s_0(t) = A\cos 2\pi f_c t\ , \quad 0 < t < T\ ,$$

and

$$s_1(t) = -A\cos 2\pi f_c t\ , \quad 0 < t < T\ .$$

Assume that the communication is through an AWGN (additive white Gaussian noise) channel, where the two-sided power spectral density of the noise is $N_0/2$.

(a). (5%) Find the average energy consumed in the transmission of one data bit. Express your answer in terms of A and T.

(b). (5%) Plot the demodulator structure for the BPSK scheme.

(c). (5%) For $A = 2$, $T = 1$ and $N_0 = 1/4$, find the bit error rate of the BPSK modulation.