

## 國立臺灣科技大學

## 八十九學年度碩士班招生考試試題

系所組別：電機工程系甲組、電機工程系乙二組

科目：工程數學

Total: 100 points.

1. Suppose that the rate of change of price  $P(t)$  with time  $t$  is proportional to the difference between supply  $S(t)$  and demand  $D(t)$ . Solve for  $P(t)$  assuming that  $D(t) = -AP(t)$  and  $S(t) = B$  for some constants  $A$  and  $B$ . (10 points)

2. Solve

$$x^3 y''' + 6x^2 y'' + 7xy' + y = 0$$

for  $y$  where  $y''' = \frac{d^3 y}{dx^3}$ ,  $y'' = \frac{d^2 y}{dx^2}$ ,  $y' = \frac{dy}{dx}$ . (10 points)

3. Solve

$$y'' - 2xy = 0$$

for  $y$  where  $y'' = \frac{d^2 y}{dx^2}$ . (10 points)

4. Simplify the matrix  $B$ :

$$B = e^A$$

where  $A = \begin{bmatrix} 2 & 1 \\ -4 & -3 \end{bmatrix}$ . (10 points)

5. Find the best approximation for the following set of data that fit the model:

$$y = mx + b. \quad (10 \text{ points})$$

$x$	-2	-1	0	1	2	3
$y$	-3.6	-0.2	1.8	5.3	8.8	11.6

6. Let  $\varphi(x, y, z) = xz + e^y z^3$ .

(a) Determine the direction from  $(1, 1, 2)$  in which  $\varphi$  has its greatest rate of change, and calculate this maximum rate of change. (5 points)

(b) Determine the rate of change of  $\varphi$  in the direction of  $\hat{a}_x + 2\hat{a}_y - \hat{a}_z$  at  $(1, 1, 2)$ . (5 points)



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7. Find the flux of  $\vec{F} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$  across the part of the sphere  $x^2 + y^2 + z^2 = 4$  lying between the planes  $z=1$  and  $z=2$ . (10 points)

8. Solve the following first order differential equation by applying the Fourier transform

$$y' - 2y = H(t)e^{-2t}, \quad -\infty < t < \infty$$

where  $H(t)$  is the unit step function (Heaviside function). (10 points)

9. Given that  $f(z) = z^2 - iz = u(x, y) + iv(x, y)$  in the complex plane.

(a) Find  $u$  and  $v$ . (5 points)

(b) Determine whether the Cauchy-Riemann equations hold or not.

(5 points)

10. Evaluate  $\oint_C \sin(z) \left( \frac{1}{(z-5)} + \frac{1}{(z-5)^2} \right) dz$ , where  $C$  is any simple closed path enclosing  $z=5$ . (10 points)

