

國立臺灣科技大學

八十九學年度碩士班招生考試試題

系所組別：電機工程系乙一組

科目：線性代數

1. (20%) Find bases of $Col(A)$ and $Nul(A)$ for the following matrix

$$A = \begin{bmatrix} 3 & 2 & 1 & 10 \\ -2 & -3 & -9 & 5 \\ 3 & 4 & 11 & -4 \end{bmatrix}.$$

2. (10%) x is in a subspace H that has a basis $B = \{b_1, b_2\}$.

Find the coordinate vector of x relative to B ,

where $b_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 \\ -7 \end{bmatrix}$, and $x = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

3. (10%) Let $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $y_1 = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$, and $y_2 = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$ and

let $T: R^2 \rightarrow R^2$ be a linear transformation that maps e_1 into

y_1 and e_2 into y_2 . Find the image of $\begin{bmatrix} 7 \\ 6 \end{bmatrix}$.

4. (10%) Find the inverse of the following matrix, if it exists.

$$\begin{bmatrix} 1 & 0 & 4 \\ 1 & 1 & 0 \\ 3 & 2 & 5 \end{bmatrix}.$$

5. (10%) Show that $\{t, \sin t, \cos 2t, \sin t \cos t\}$ is a linearly independent set of function defined on R .

6. (10%) Show that if a vector space V has a basis of n vectors, then every basis of V must consist of exactly n vectors.

7. (10%) Let $A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 1 & 4 & -3 & 0 \\ -1 & -2 & 0 & -3 \end{bmatrix}$. Find the determinant and

the characteristic equation of A .

8. (10%) Find the eigenvalues and bases for their corresponding eigenspaces for the matrix A in problem 7.

9. (10%) Find the orthogonal projection of $y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ onto $u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$.

