國立臺灣科技大學
八十九學年度碩士班招生考試試題

系所組別： 工業管理系甲組
科　目： 作業研究

1. Each family in Taiwan is classified as living in a city, rural, or suburban location. During a given year, 15% of all city families move to a suburban location, and 5% move to a rural location; also, 6% of all suburban families move to a city location, and 4% move to a rural location; finally, 4% of all rural families move to a city location, and 6% move to a suburban location.
   a. If a family now lives in a city, what is the probability that it will live in a rural area two years from now (10%)?
   b. Suppose that at present, 40% of all families live in a city area, 35% live in a suburban, and the rest live in a rural area. Two years from now, what percentage of families in Taiwan will live in a city area (15%)?

2. A company has five sales representatives available for assignment to three sales districts. The sales in each district during the current year depend on the number of sales representatives assigned to the district and on whether the national economy has a bad or good year, as indicated in the following table. In the Sales column for each district, the first number represents sales if the economy had a bad year, and the second one represents the sales for a good year. There is a 0.7 chance that the national economy will have a bad year, and 0.3 chance for a good one. Furthermore, there are at most three sales representatives can be assigned to a district. Use dynamic programming to determine an assignment of sales representatives to districts that maximizes the company expected sales (25%).

No. of sales rep. assigned to district	Sales (Millions) District 1	District 2	District 3
0	$1, $4	$2, $5	$3, $4
1	$2, $6	$4, $6	$5, $5
2	$3, $7	$5, $6	$6, $7
3	$4, $8	$6, $6	$7, $7

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3. Consider the following linear programming problem:

$$\text{Maximize } z = 2x_1 + x_2 + x_3$$

$$\begin{aligned} \text{Subject to} \quad & x_1 + x_3 \le 1 \\ & x_2 + x_3 \le 2 \\ & x_1 + x_2 \le 3 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

It is given that

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

a. Show that the basic solution with basic variables x_1, x_2, and x_3 is optimal. Find the optimal solution (10%).
b. Write down the dual to this problem and find its optimal solution (10%).
c. Show that if the right-hand-side of each constraint of the original problem is multiplied by a nonnegative constant k, then the new optimal solution is obtained simply by multiplying the value of each variable in the original optimal solution by k (5%).

4. A cellular phone company is going to open a new branch at a mall, and the management is attempting to determine the number of salespersons to hire. Based on an analysis of mall traffic, the company estimates that customers will arrive at the store at the rate of 10 per hour, and from past experience at its other branches, the company knows that a salesperson can serve an average of six customers per hour. How many salespersons should the company hire in order to uphold a company policy that on the average the probability of a customer having to wait for service be no more than 30% (25%)?