國立臺灣科技大學

八十九學年度碩士班招生考試試題

系所組別： 資訊管理系甲組

科　目： 離散數學

1. (15%)The **Fibonacci numbers** are defined by

$$F_1 = 1, F_2 = 1, \text{ and if } n > 2 \text{ then } F_n = F_{n-1} + F_{n-2}$$

Use mathematical induction to prove that $F_n \leq 2^n$, for any positive integer n.

2. (10%)In a binary tree in which each vertex has 0 or 2 children, prove that the number of terminal vertices is one more than the number of internal vertices.

3. (10%)The following information was found about the residents of a certain retirement community:
   38 played golf.
   21 played tennis.
   56 played bridge.
   8 played golf and tennis.
   17 played golf and bridge.
   13 played tennis and bridge.
   5 played golf, tennis, and bridge.
   72 did not play golf, tennis, or bridge.
   How many residents are there in this retirement community?

4. (a) (5%)If graph $G = (V, E)$ is a **connected graph** with $|E| = 17$ and $degree(v) \geq 3$ for all $v \in V$, what is the maximum value for $|V|$? (the degree of v is the number of edges connected to v)

   (b) (5%)How many vertices and edges does the **complete bipartite graph** $K_{m,n}$ have?

   (c) (5%)The **connected graph** $G = (V, E)$ has 30 edges. What is the maximum value that $|V|$ can have?

5. (17%) Use the Quine–McCluskey method to find a minimal expansion equivalent to $xyz+x\bar{y}z+\bar{x}y\bar{z}+\bar{x}\bar{y}z+\bar{x}\bar{y}\bar{z}$.

6. (17%) Solve the congruence $2x \equiv 7 \pmod{17}$.

7. (16%) Show that the "greater than or equal" is a partial ordering on the set of integers.