

國立臺灣科技大學
八十九學年度碩士班招生考試試題

系所組別：資訊管理系丙組
科目：作業研究

1. Consider the following linear programming problem:

$$\text{Maximize } Z = \alpha X_1 + 75X_2$$

$$\begin{aligned} \text{subject to } & X_1 + 3X_2 \leq \beta \\ & 2X_1 + \gamma X_2 \leq 6 \\ & X_1, X_2 \geq 0 \end{aligned}$$

The final simplex tableau is:

C_B	Basis	X_1	X_2	X_3	X_4	b
75	X_2	0	1	$2/5$	$-1/5$	$2/5$
α	X_1	1	0	$-1/5$	$3/5$	$14/5$
	$\bar{C}_j$	θ_1	θ_2	θ_3	θ_4	$Z = 254$

- What are the values of α , β , γ , θ_1 , and θ_3 ? (10%)
 - Give the range of α within which the current optimal solution is still optimal. (5%)
 - Give the range of β within which the current optimal solution is still optimal. (5%)
2. A device is subject to shocks which occur according to a stationary Poisson process $\{N_t; t \geq 0\}$ with rate $\lambda = 3$.
- Find $P\{N_6 = 4, N_{20} = 12\}$. (5%)
 - The device can fail only due to a shock, and the probability that a given shock causes failure is $p = 0.3$ independent of the number and times of previous shocks. Let T be the time of failure. Compute the expectation of T . (5%)
 - Find $E[N_{t+s}|N_t]$. (5%)
3. Consider the following nonlinear programming problem:

$$\text{minimize } -X_1 - X_2 + X_2^2$$

$$\begin{aligned} \text{subject to } & X_1 + X_2 \leq 1 \\ & X_1 \geq 0 \\ & X_2 \geq 0 \end{aligned}$$

- Write down the Kuhn-Tucker conditions for this problem. (5%)
 - Solve this problem. Is this the global minimal solution? (5%)
4. Consider a Markov chain with three states a, b , and c . Suppose that the Markov and stationary assumptions hold. Given the initial state distribution $P\{X_0 = i\} = 1/3, i = a, b$, and c and the one-step transition matrix

$$P = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/4 & 1/4 & 2/4 \\ 3/5 & 1/5 & 1/5 \end{bmatrix},$$

find the following probabilities.



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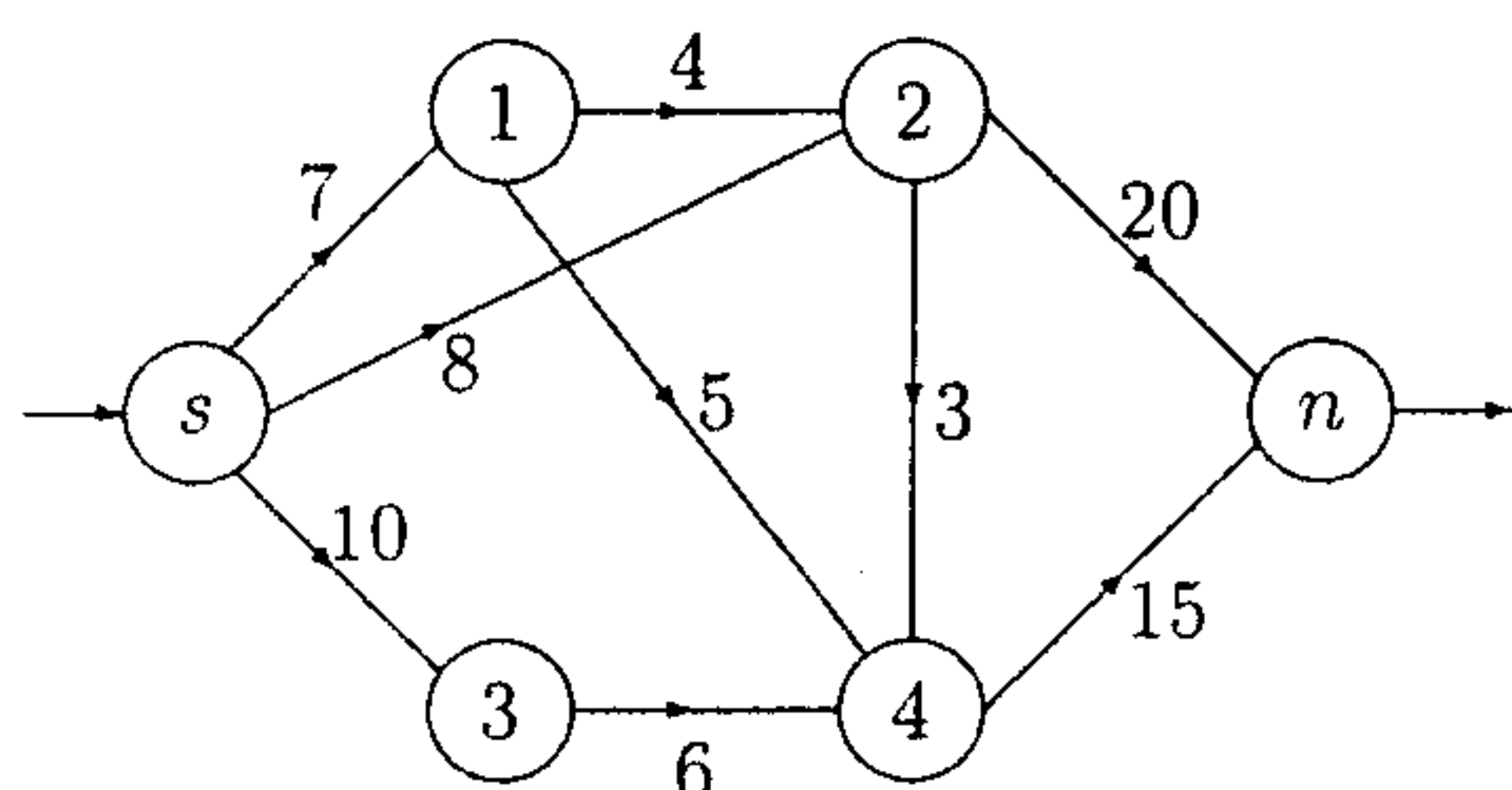
(a) $P\{X_2 = a | X_0 = a\}$ (5%)

(b) $P\{X_2 = a\}$ (5%)

5. Consider the following network, where the numbers of the arcs represent their capacities.

(a) Compute the maximal flow from s to n and specify how this flow is routed through various arcs. (5%)

(b) Identify the minimal cut. (5%)



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6. (10 points) Show that the inner product of two vectors, u and v , is less than or equal to the product of the two norms, $\|u\|$ and $\|v\|$.
7. (15 points) Let A and B be $n \times n$ matrices. If B is similar to A , then show that the two matrices both have the same eigenvalues.
8. (10 points) Let Y be a subspace of R^n . Please define the orthogonal complement of Y .

