

國立臺灣科技大學

九十學年度碩士班招生考試試題

系所組別：工程技術研究所自動化及控制學程甲組、工程技術研究所自動化及控制學程丙組

科目：工程數學

總分 100 分

1. Consider the circuit of Fig. 1 with system differential equation as

$$y''(t) + 7y'(t) + 12y(t) = 4x(t) + 2x'(t)$$

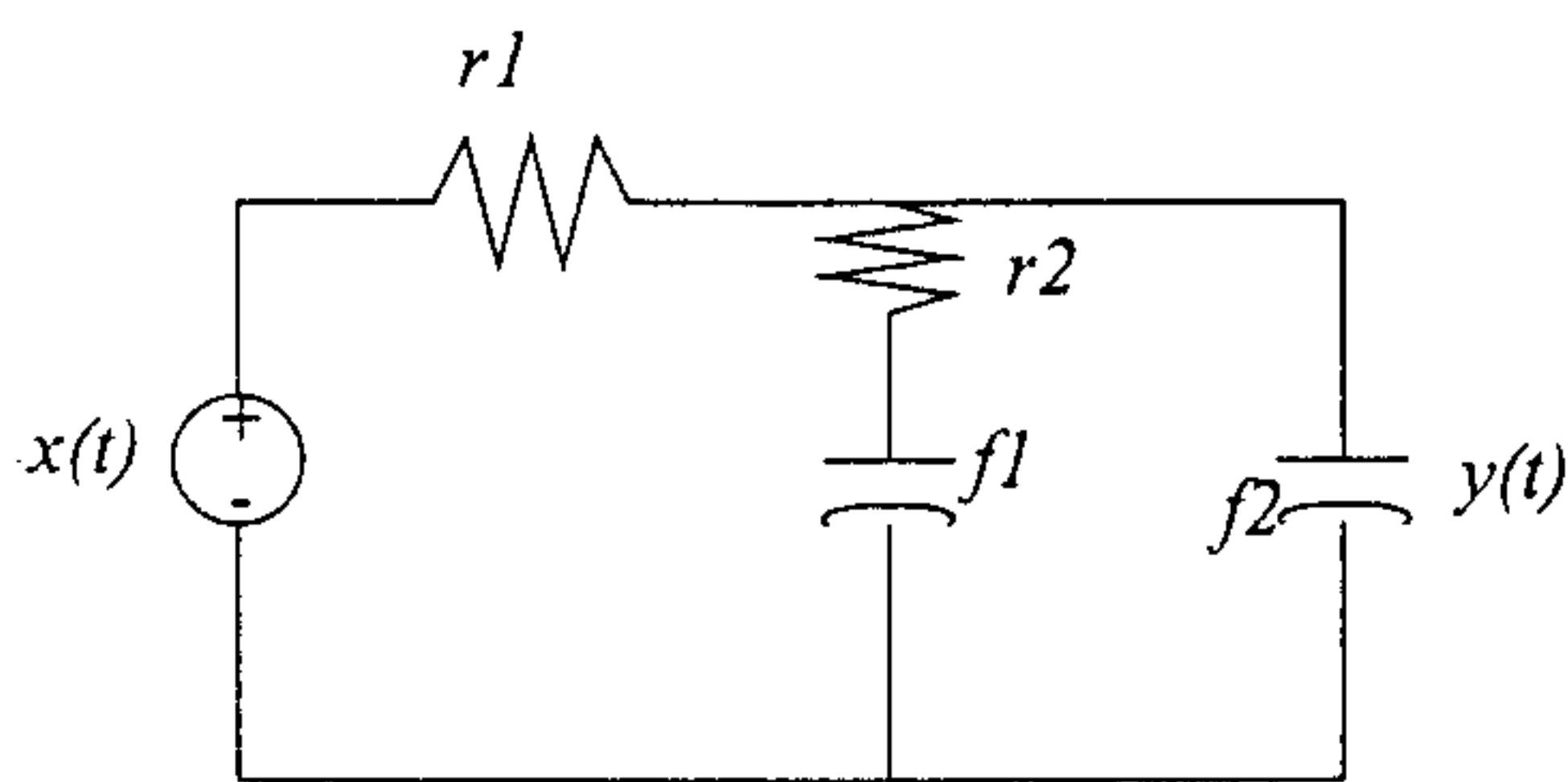


Fig. 1

Please solve the following by Laplace Transform

(a) Find the impulse response $h(t)$, i.e. solve the output $y(t)$ as $x(t) = \delta(t)$,

$$\text{where } \delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases} \quad (10\%)$$

(b) Find the zero state output $y_{z.s.}(t)$ by the convolution integral of $h(t)$ and $x(t)$, ($y_{z.s.}(t) = h(t) * x(t)$),

$$\text{when } x(t) = t u(t), \text{ where } u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \quad (10\%)$$

2.(a) Determine the Fourier Transform of the function

$$f(t) = \frac{10}{t^2 + 1} \quad (10\%)$$

(b) Determine the complex exponential Fourier series coefficient c_n for the periodic function

$$f(t) = e^t, 0 \leq t \leq 1, \text{ and which has the period } T=1, \text{ shown in Fig. 2,} \quad (5\%)$$

and please plot the complex Fourier spectrum ($|c_n|$ versus $n\omega_0$) (5%)

$$(\text{Note : } e^1 = 2.72, \frac{1.72}{\sqrt{1+4\pi^2}} = 0.27, \frac{1.72}{\sqrt{1+16\pi^2}} = 0.136)$$

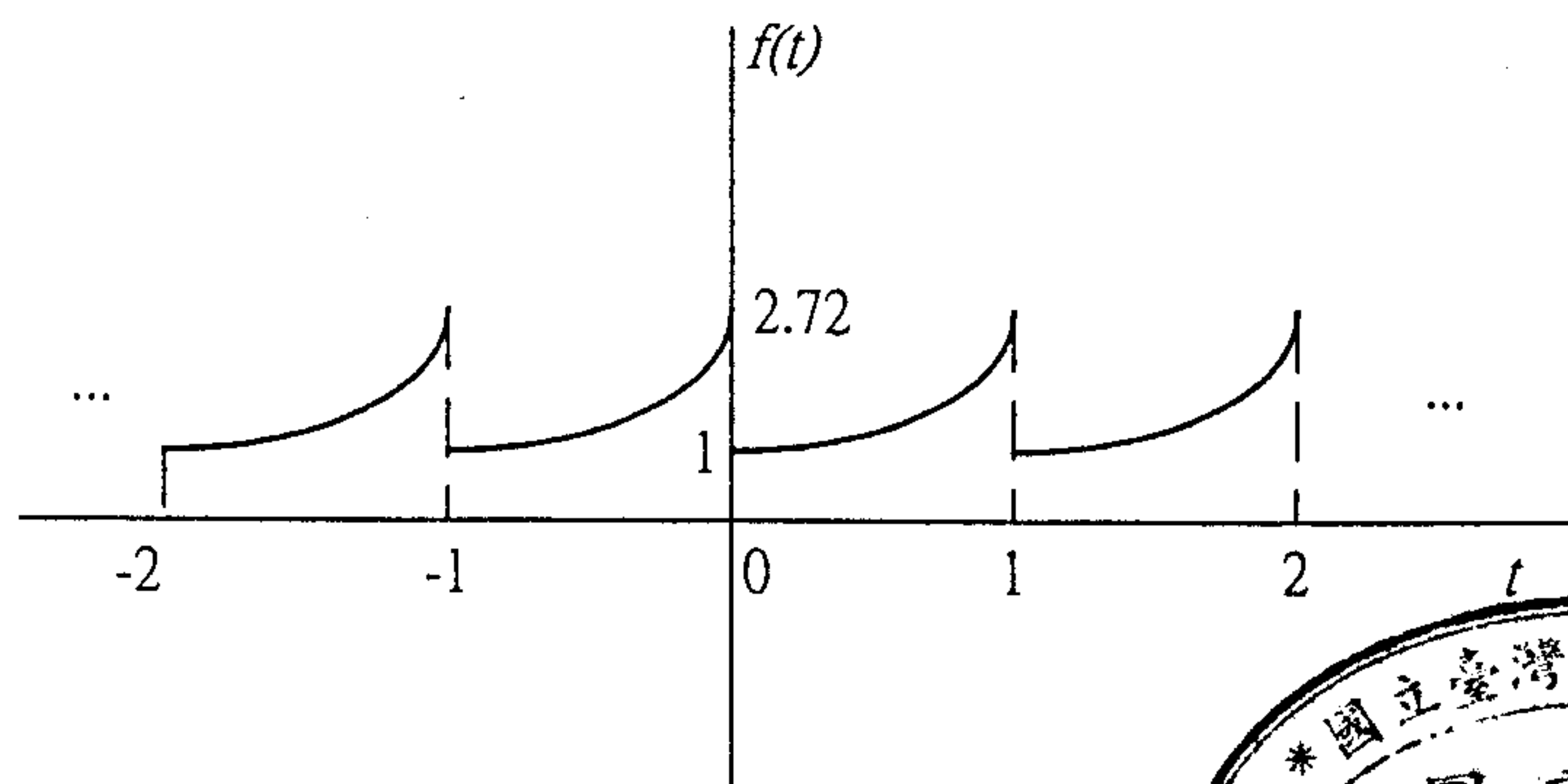


Fig. 2

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3. Solve the following differential equations:

$$(a) \frac{dy}{dx} = \frac{1}{e^y - x} \quad (10\%)$$

$$(b) (x-1)y'' - xy' + y = 0 \quad (10\%)$$

4. A spring mass system shown in Fig.3 with the motion equations as

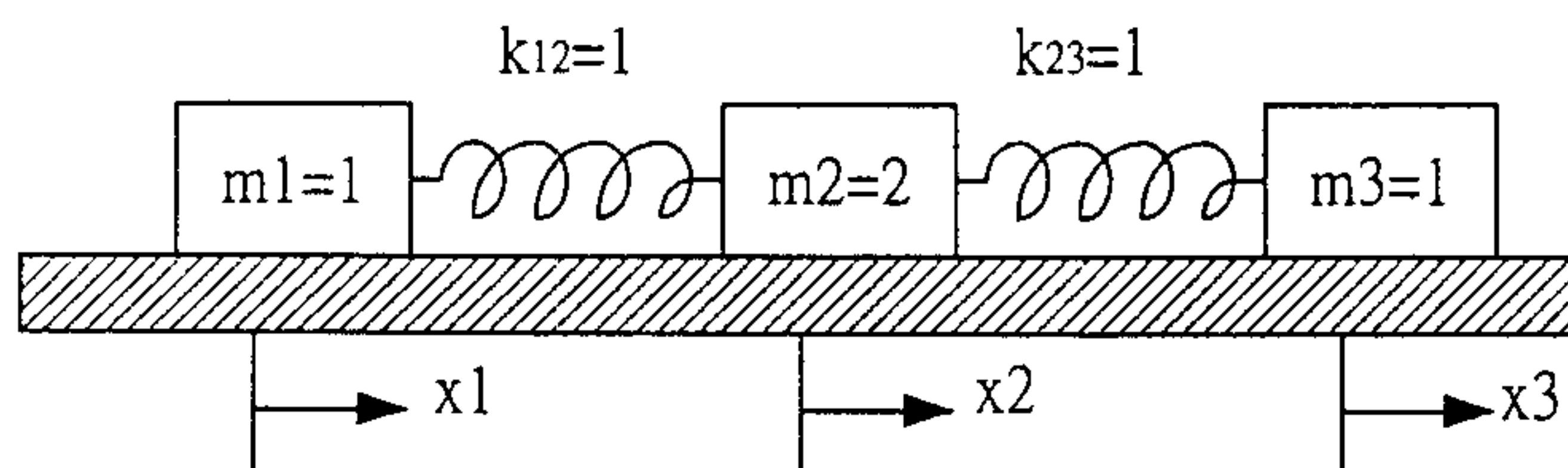


Fig. 3

where m_1, m_2, m_3 are mass point moving in the x-direction and x_1, x_2, x_3 are the position, which values are shown in the figure.

(a) Please derive the motion equations for m_1, m_2, m_3 , and then transfer the motion equations into a matrix form as $d^2\mathbf{X}/dt^2 = \mathbf{A}\mathbf{X}$, and find the eigen values and eigen vectors of $\mathbf{A}$ (10%)

(b) Find the possible eigen frequency ω , and the corresponding values of x_1, x_2, x_3 . (10%)

5. z is a complex variable.

(a) Locate and classify the singularity for the following two functions:

$$f_1(z) = \frac{\ln(z-2)}{z^2(z^2+4)} \quad (5\%)$$

$$f_2(z) = \frac{\sin(\sqrt{z})}{\sqrt{z}} \quad (5\%)$$

(b) Evaluate $\oint_C \frac{(2z+1)}{z^3 - iz^2 + 6z} dz$ with C the circle of radius $\frac{1}{3}$ about the point $(0,3)$. (10%)



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