國立臺灣科技大學
九十學年度碩士班招生考試試題
系所組別： 工程技術研究所自動化及控制學程乙組
科　目： 作業研究

OPERATIONS RESEARCH

(/or/acc2001.ctx)

1. The optimal solution of the problem

$$\text{maximize } cx \quad \text{subject to} \quad Ax = b, \qquad x \geq 0$$

with

$$A = \begin{bmatrix} 8 & 5 & 1 & 0 & 0 \\ 8 & 6 & 0 & 1 & 0 \\ 8 & 7 & 0 & 0 & 1 \end{bmatrix},$$

$$b = \begin{bmatrix} 32 \\ 33 \\ 35 \end{bmatrix}$$

$$c = [c_1, c_2, c_3, c_4, c_5] = [\ 1 \quad 1 \quad 0 \quad 0 \quad 0\ ]$$

is $[0, 5, 7, 3, 0]^T$.

(a) (10%) Construct the final optimal tableau.

(b) (5%) Find the range of c_2 such that the current basis is still optimal.

(c) Solve the same linear program,

(1) (5%) if c changes to $[2, 1, 0, 0, 0]$.

(2) (5%) if b changes to $[30, 33, 35]^T$.

(3) (5%) if the 1st column of A changes to $[7, 7, 7]^T$.

You may need the following information:

$$\begin{bmatrix} 5 & 1 & 0 \\ 6 & 0 & 1 \\ 7 & 0 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0 & 1/7 \\ 1 & 0 & -5/7 \\ 0 & 1 & -6/7 \end{bmatrix}.$$

2. Consider the linear programming problem

$$\text{Maximize } Z = 3x_1 + 4x_2 + 2x_3$$

$$\begin{array}{lrcrcrcl} \text{subject to} & -x_1 & + & x_2 & + & x_3 & = & 4 \\ & -x_1 & + & x_2 & - & x_3 & \leq & 6 \\ & x_1 & \leq & 0, & & x_2 & \geq & 0. \end{array}$$

(a) (10%) Give the dual problem.

(b) (5%) Solve the dual problem by graphical method.

(c) (10%) Solve the primal problem by complementary slackness conditions.

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3. In the tableau of the minimization problem below, the values of six constants $\alpha_1, \alpha_2, \alpha_3, \beta, \rho_1, \rho_2$ are unknown (assume there are no artificial variables):

Basis	x_1	x_2	x_3	x_4	x_5	x_6	RHS
x_3	4	α_1	1	0	α_2	0	β
x_4	-1	-5	0	1	-1	0	2
x_6	α_3	-3	0	0	-4	1	3
$\bar{c}_j$	ρ_1	ρ_2	0	0	3	0	

$$x_1, x_2, \ldots, x_6 \geq 0$$

State minimum restrictions on the six unknowns which would make the following statements true about the given tableau:

(a) (5%) The current solution is the unique optimal solution.

(b) (5%) The current solution is feasible but the problem is unbounded.

(c) (5%) The current solution is a degenerate basic feasible solution.

4. (10%) Find the optimal assignment of four jobs (J_i) to four machines (M_i) when the cost of assign is given in the following table.

	J_1	J_2	J_3	J_4
M_1	10	9	8	7
M_2	3	4	5	6
M_3	2	1	1	2
M_4	4	3	5	6

5. (20%) Consider the following directed network where S is the source node, N is the sink node, and the numbers $[i, j]$ along each arc has capacity i and the current flow is j. Find the maximum flow from S to N. Identify the minimal cut.