

國立臺灣科技大學

九十學年度碩士班招生考試試題

系所組別：電機工程系甲組、電機工程系乙二組

科 目：工程數學

(共十題；滿分 100 分)

1. Find the general solution to

$$y''' - y'' - 8y' + 12y = 7e^{2x}. \quad (10\%)$$

2. Find the general solution to

$$x^2 y'' + 2xy' - 12y = \sqrt{x} \text{ for } x > 0. \quad (10\%)$$

3. Consider the inner product space $C[0,1]$ with inner product defined by

$$\langle f, g \rangle = \int_0^1 f(x)g(x)dx$$

Let S be the subspace spanned by the vectors 1 and $2x-1$. Find the best least squares approximation to \sqrt{x} by a function from the subspace S . (10%)

4. Solve the system of equations

$$y_1' = 3y_1 + 4y_2$$

$$y_2' = 3y_1 + 2y_2$$

where $y_1(0) = 6$ and $y_2(0) = 1$. (10%)

5. Find A^k where k is a positive integer and

$$A = \begin{pmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{pmatrix} \quad (10\%)$$



55

國立臺灣科技大學

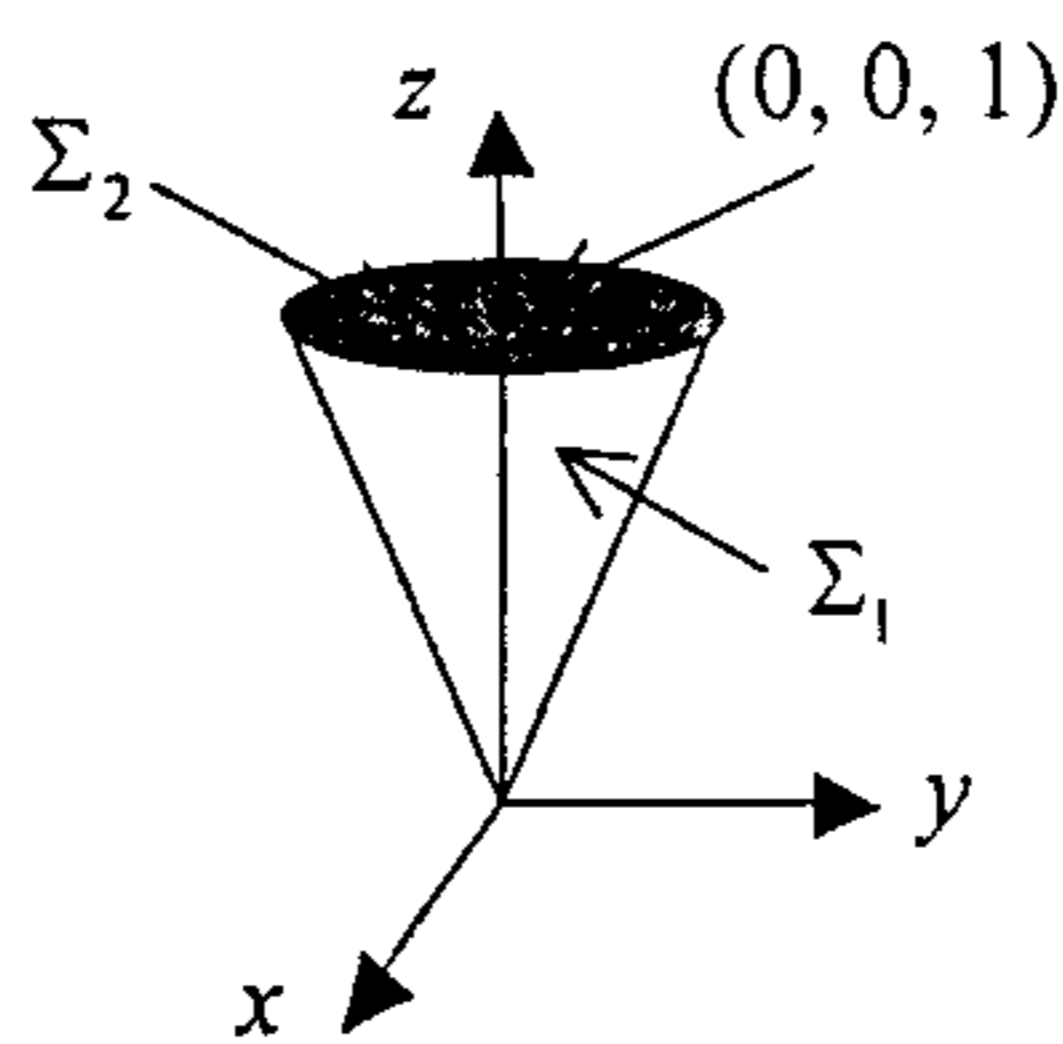
九十學年度碩士班招生考試試題

系所組別：電機工程系甲組、電機工程系乙二組

科目：工程數學

6. For $\mathbf{F}(x, y, z) = 3x^2\hat{a}_x - z^2\hat{a}_y - 2yz\hat{a}_z$.(a) Find a potential function for $\mathbf{F}(x, y, z)$. (7%)(b) Evaluate the line integral of $\mathbf{F}(x, y, z)$ over a piecewise-smooth curve from $(2, 1, 3)$ to $(-3, 5, -1)$. (3%)

7. Let Σ be the closed surface consisting of the surfaces Σ_1 of the cone $z^2 = x^2 + y^2$ for $0 \leq x^2 + y^2 \leq 1$ and the flat cap Σ_2 consisting of the disk $x^2 + y^2 \leq 1, z = 1$, as shown in the following figure. Illustrate Gauss's Divergence Theorem by separately computing both sides of the equation for a vector field $\mathbf{F}(x, y, z) = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$. (10%)



8. Determine the complex Fourier series of the sawtooth function f of period 5 defined by (10%)

$$f(t) = \frac{2}{5}t \quad \text{for } 0 < t < 5$$

$$f(t \pm 5k) = f(t), \quad k = 1, 2, 3, \dots$$

9. Find the Fourier transform of the following function (10%)

$$f(t) = 4e^{-3t^2} \sin(2t)$$

10. Evaluate $\int_C |z|^2 dz$, where C is the straight line segment from -2 to i in the complex plane. (10%)



56