國立臺灣科技大學

九十學年度碩士班招生考試試題

系所組別： 電機工程系乙一組

科　目： 線性代數

(共八題；滿分一百分)

1. (10%) A parallelogram has two incident sides extending from (0, 1, −2) to (1, 2, 2) and from (0, 1, −2) to (1, 4, 1). Find the area of this parallelogram.

2. (10%) One corner of a rectangular parallelepiped is at (−1, 2, 2) and three incident sides extend from this point to (0, 1, 1), (−4, 6, 8), and (−3, −2, 4). Find the volume of this parallelepiped.

3. (10%) Find the determinant and the inverse of the following matrix, if it exists.

$$\begin{bmatrix} -3 & 4 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix}$$

4. (20%) (Principal axes theorem) Consider the following equation

$$17x_1^2 - 30x_1x_2 + 17x_2^2 = 128$$

Rotate the coordinate axes to remove the x_1x_2 term. Name the conic and give its equation in the rotated coordinate system (Hint: Express $X^T = [x_1 \quad x_2]$ in terms of the new coordinate vector $Y^T = [y_1 \quad y_2]$).

5. (15%) Find bases of *Col*(*A*) and *Nul*(*A*) for the following matrix

$$A = \begin{bmatrix} 3 & 0 & 1 & 1 \\ 2 & 4 & 2 & 1 \\ 1 & 8 & 3 & 1 \end{bmatrix}.$$

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6. (10%) Let x have the coordinate $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ in a subspace $\boldsymbol{H}$ that has a basis $\boldsymbol{A} = \{\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}\}$. Find the coordinate vector of x relative to another subspace $\boldsymbol{G}$, which has a basis $\boldsymbol{B} = \{\begin{bmatrix} 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ -7 \end{bmatrix}\}$.

7. (10%) Let $\{x_1, x_2, \ldots, x_p\}$ be a set of vectors in R^m. If $p>m$, show that $\{x_1, x_2, \ldots, x_p\}$ is linearly dependent.

8. (15%) Let $A = \begin{bmatrix} 3 & 2 & -3 \\ 0 & 1 & 0 \\ 2 & 2 & -2 \end{bmatrix}$. Find the eigenvalues and bases for their corresponding eigenspaces for the matrix A.

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