國立臺灣科技大學

九十學年度碩士班招生考試試題

系所組別： 資訊工程系、電子工程系甲組、電機工程系丙組

科　目： 離散數學

1. (a) Let $\oplus$ denote the symmetric difference operation. For any set **A**, what is $\mathbf{A} \oplus \mathbf{A}$? $\mathbf{A} \oplus \mathbf{\Phi}$ (where $\mathbf{\Phi}$ denotes the empty set)? (5%)

   (b) Given infinite series $\Sigma\, \mathbf{a_n}$ and $\Sigma\, \mathbf{b_n}$. Is the following statement

   "$\Sigma\, \mathbf{a_n}$ converges if and only if $\Sigma\, \mathbf{b_n}$ converges"

   Logically equivalent to the statement

   "$\Sigma\, \mathbf{a_n}$ diverges if and only if $\Sigma\, \mathbf{b_n}$ diverges" ?

   Explain your answer. (5%)

   (c) Write the negation of the following statement

   "For every $\varepsilon > 0$, there exists $N > 0$, such that $x > N$ implies $| f(x) - L | < \varepsilon$". (5%)

2. Given recurrence relation

   $S_0 = 1$ and $S_n = 2\, S_{n-1} + n$

   (a) Find an explicit formula for S_n. (8%)

   (b) Prove your answer in (a) by mathematical induction. (7%)

3. Let S = {a, b, c, d} and T = {1, 2, 3, 4, 5, 6, 7}

   (a) How many one-to-one functions are there from T into S? (3%)

   (b) How many one-to-one functions are there from S into T? (3%)

   (c) How many functions are there from S into T? (4%)

4. Let N be the set of positive integers with the usual ordering, i.e., $1 < 2 < 3 < 4 < \ldots$.

   (a) Define relation R on $N \times N$ as

   $R = \{((a, b), (c, d)) \mid a \leqq c \text{ or } b \leqq d\}$

   Is R a partial order? Total order? Or neither? Explain your answer. (5%)

   (b) Define relation S on $N \times N$ as

   $S = \{((a, b), (c, d)) \mid$ If $a < c$ or
   If $a = c$ and $b < d$ or
   If $a = c$ and $b = d$ $\}$

   Is S a partial order? Total order? Or neither? Explain your answer. (5%)

5. Please find an addition chain for the integer 90. (10%)

6. Please construct a grammar for the language $L = \{a^i b^{2i} | i \geq 1\}$. (10%)

7. Please construct an optimal prefix code for the symbols A, O, Q, U, Y, and Z that occur (in a given sample) with frequencies 20, 28, 4, 17, 12, and 7, respectively. (10%)

8. Show that a circuit and the complement of any spanning tree must have at least one edge in common. (10%)

9. Show that the order of any subgroup of a finite group divides the order of the group. (10%)

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