國立臺灣科技大學
九十學年度碩士班招生考試試題
系所組別：資訊工程系、電子工程系甲組、電機工程系丙組
科　　目：資料結構

1. Table 1 contains 8 jobs and their anticipated run time. The scheduler of an operating system schedules jobs based on their anticipated run time with **shortest-job-first** algorithm. Please use "heap" to implement the priority scheduler.
   (a) Draw the heap after each job is inserted into the scheduler following the sequence shown in Table 1 (first, insert *J1* then *J2, J3,* ...). (8%)
   (b) What make the heap the preferred representation of priority queue? Compare the insertion time and delete time of the implementation of priority queue using unordered array, sorted link-list and heap. (8%)

Table 1

Job Identifier	Anticipated Run Time (ms)
J1	25
J2	57
J3	48
J4	37
J5	12
J6	92
J7	86
J8	33

2. Given a binary tree which inorder transversal sequence: C E A D B and postorder traversal sequence E C D B A.
   (a) Draw the internal memory representation of the binary tree using array representation. (8%)
   (b) Draw the corresponding fully threaded binary tree. (6%)

3. Algorithm A has uniform time complexity $\alpha * n^2$, where α is a constant and n is the size of the data set to be processed. The Superman Company makes a computer that can execute Algorithm A on a problem of size n = 50 in 100 ms (= 0.1 sec). The Ruby Company has just built a computer on which the time for an operation is 1/10 (one tenth) that of the same operation on the Superman Company's machine. Using Algorithm A, what is the largest problem size n that will execute in 100 ms on the Ruby Company's machine? (8%)

4. Sorted list A has n elements; sorted list B has m elements.
   (a) What is the **minimum** number of comparisons between elements of A and elements of B that will be required to merge Lists A and B to produce sorted list C ? (Please describe the condition for the minimum number of comparisons) (6%)
   (b) What is the **maximum** number of comparisons between elements of A and elements of B that will be required to merge Lists A and B to produce sorted list C ? (Please describe the condition for the maximum number of comparisons) (6%)

(To be continued)

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5. A totally connected graph has a direct path or connection between any two distinct nodes. A *clique* of a graph is a totally connected subgraph. A *maximal clique* is not included in any other clique. For the graph shown in Figure 1, illustrate the stacking process in detail to find the maximal cliques by means of the following procedure:

<Input> X : a clique, expressed as a set of nodes (possibly empty).
Y : a set of nodes representing a graph or subgraph.
<Output> The set of all maximal cliques in Y that include X.

PROCEDURE $cliques(X, Y)$;
BEGIN
Form $Y - X$;
IF a node y in $Y - X$ is connected to every element of X,
THEN return $cliques(X \cup \{y\}, Y) \cup cliques(X, Y - \{y\})$
ELSE return X.
END

(14%)

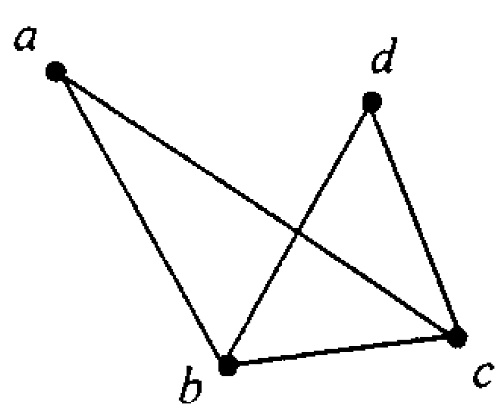

Figure 1.

6. Work through Binary search and Fibonacci search on an ordered file with keys (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16) and determine the number of key comparisons made while searching for the keys 2, 10, and 15. (For Fibonacci search, we need three Fibonacci numbers $\mathbf{F}_5 = 5$, $\mathbf{F}_6 = 8$, and $\mathbf{F}_7 = 13$) (12%)

7. Suppose that we have a hash table with n slots, with collisions resolved by chaining, and suppose that n keys are inserted into the table. Each key is equally likely to be hashed to each slot. Let M be the maximum number of keys in any slot after all the keys have been inserted.

(a) Argue that the probability Q_k that k keys hash to a particular slot is given by

$$Q_k = \left(\frac{1}{n}\right)^k \left(1 - \frac{1}{n}\right)^{n-k} \binom{n}{k}.$$ (4%)

(b) Let P_k be the probability that $M = k$, that is, the probability that the slot containing the most keys contains k keys. Show that $P_k \le nQ_k$. (4%)

(c) Use Stirling's approximation to show that $Q_k < e^k / k^k$. (4%)

(d) Show that there exists a constant $c > 1$ such that $Q_{k_0} < 1/n^3$ for $k_0 = c\log_2 n / \log_2 \log_2 n$. Conclude that $P_{k_0} < 1/n^2$ for $k_0 = c\log_2 n / \log_2 \log_2 n$. (6%)

(e) Argue that the expected value of M:

$$E[M] \le \Pr\left\{M > \frac{c\log_2 n}{\log_2 \log_2 n}\right\} \cdot n + \Pr\left\{M \le \frac{c\log_2 n}{\log_2 \log_2 n}\right\} \cdot \frac{c\log_2 n}{\log_2 \log_2 n}.$$

Conclude that $E[M] = O(\log_2 n / \log_2 \log_2 n)$. (6%)

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