

## 國立臺灣科技大學

## 九十學年度碩士班招生考試試題

系所組別：工業管理系甲組、工業管理系乙組、工業管理系丙組、工業管理系丁組

科目：統計學

共 5 題 2 頁,請寫出計算過程,只寫答案不能獲得所有配分

1. Suppose a random sample of size  $n$  is drawn from the two parameter normal pdf,

$$f_Y(y; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}, -\infty < y < \infty; -\infty < \mu < \infty; \sigma^2 > 0$$

Use the method of maximum likelihood to find formulas for  $\hat{\mu}$  and  $\hat{\sigma}^2$ .Is  $\hat{\sigma}^2$  unbiased for  $\sigma^2$ ? If not, what function of  $\hat{\sigma}^2$  does have an expected value equal to  $\sigma^2$ ? (20%)2. Let  $Y_1, Y_2, \dots, Y_n$  be a random sample from the uniform pdf defined over theinterval  $[0, \theta]$ . We know that  $\hat{\theta}_1 = \frac{2}{n} \sum_{i=1}^n Y_i$  and  $\hat{\theta}_2 = \frac{n+1}{n} Y_{\max}$  are both unbiased for  $\theta$ ,where  $Y_{\max} = \max\{Y_1, Y_2, \dots, Y_n\}$ . Which estimator is more efficient? (15%)3. A random sample of size  $n=8$  is drawn from the uniform pdf,  $f_Y(y; \theta) = \frac{1}{\theta}, 0 \leq y \leq \theta$ 

for the propose of testing

$$H_0: \theta = 2.0$$

versus

$$H_1: \theta < 2.0$$

at the  $\alpha = 0.10$  level of significance. Suppose the decision rule is to be based on  $Y'_8$ , the largest order statistic, that is  $Y'_8 = \max\{Y_1, Y_2, \dots, Y_8\}$ . What would be the probability of committing a Type II error when  $\theta = 1.7$ ? (15%)

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4. (20 points)

Consider a random variable  $X$  with density function  $f(x) = \begin{cases} \frac{1}{5} & 0 \leq x \leq 5 \\ 0 & \text{elsewhere} \end{cases}$ .

(a) (10 points) Find  $\mu = E(X)$  and  $\sigma^2 = E(X - \mu)^2$ .(b) (10 points) Demonstrate that Chebyshev's theorem holds for  $k=1.5$ .(Hint:  $P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - 1/k^2$ .)

5. (30 points)

An entrepreneur would like to determine whether it would be profitable to establish a gardening service in a local suburb. The entrepreneur believes that there are four possible levels of demand for this service:

 $E_1$ : very low demand; 1% of the households would use the service. $E_2$ : low demand; 5% of the households would use the service. $E_3$ : moderate demand; 10% of the households would use the service. $E_4$ : high demand; 25% of the households would use the service.

On the basis of past experience in the other suburbs, the entrepreneur assigns the following prior probabilities to these demand levels:

 $P(E_1) = 0.2, P(E_2) = 0.5, P(E_3) = 0.2,$  and  $P(E_4) = 0.1$ .

Before the decision is made, a random sample of 5 households is selected and only 1 would use the service.

(a) (10 points) Given that the true demand is moderate, what is the probability that the above random sample would occur?

(b) (10 points) Under the prior probabilities, what is the probability that the above random sample would occur?

(c) (10 points) Revise the prior probabilities in light of this sample information.



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