國立臺灣科技大學
九十學年度碩士班招生考試試題

系所組別： 工業管理系甲組
科　目： 作業研究

1. ABC Company is contracted for the next 4 years to supply aircraft engines at the rate of four engines a year. Available production capacity and production costs vary from year to year. ABC can produce five engines in year 1, six in year 2, three in year 3, and five in year 4. The corresponding production costs per engine over the next 4 years are \$300,000, \$330,000, \$350,000, and \$420,000, respectively. ABC Company can choose to produce more than it needs in a certain year, in which case the engines must be properly stored until they are shipped to customers. The storage cost per engine charged at the end of year also varies from year to year, and is estimated to be \$20,000 for year 1, \$30,000 for year 2, \$40,000 for year 3, and \$50,000 for year 4. Currently, at the beginning of year 1, ABC has one engine ready for shipping. Use dynamic programming to find an optimal production plan for ABC (20%).

2. Consider the following linear programming problem and its optimal tableau after the application of Big M method:

$$\begin{aligned} \text{Maximize } Z &= 3x_1 + x_2 \\ \text{subject to } & 2x_1 + x_2 \le 4 \\ & 3x_1 + 2x_2 \ge 6 \\ & 4x_1 + 2x_2 = 7 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Basis \ x_j	Z	x_1	x_2	x_3	x_4	a_1	a_2	RHS
Z	1	0	0	0	1	$M-1$	$M+3/2$	9/2
x_3	0	0	0	1	0	0	$-1/2$	1/2
x_2	0	0	1	0	-2	2	$-3/2$	3/2
x_1	0	1	0	0	1	-1	1	1

where x_3 is the slack for the first constraint, x_4 is the surplus variable for the second constraint, a_1 and a_2 are the artificial variables for the second and the third constraints, respectively; and $x_3 \ge 0$, $x_4 \ge 0$, $a_1 \ge 0$, $a_2 \ge 0$.

a. Find the dual to this problem and its optimal solution (10%).
b. Find the range of values for the right-hand side of the third constraint for which the current basis remains optimal (10%).
c. Find the new optimal solution if the right-hand side of the third constraint is 15/2 (10%).

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3. A fair coin is tossed, and the result is shown to player 1. Player 1 must then decide whether to pass（放棄）or bet（賭）. If player 1 passes, he must pay player 2 $1. If player 1 bets, player 2 (who does not know the result of the coin toss) may either fold or call the bet. If player 2 folds, she pays player 1 $1. If player 2 calls and the coin comes up heads, she pays player 1 $2; if player 2 calls and the coin comes up tails, player 1 must pay her $2. Formulate this as a two-person zero-sum game. Then graphically determine the value of the game and each player's optimal strategy. (*Hint:* Player 1's strategies may be represented as follows: PP (pass on heads and pass on tails), PB (pass on heads and bet on tails), BP (bet on heads and pass on tails), and BB (bet on heads and bet on tails). Player 2 simply has the two strategies, call and fold.) (30%)

4. Let $y_1, y_2, \ldots, y_m$ be 0-1 variables in some integer programming problem. Write down a linear constraint or constraints that could be added to the problem in order to enforce the following conditions. (Each of the following is independent.)
(a) At least one of the following constraint must hold: (7%)
$$-3x_1 + 2x_2 - x_3 \geq 0$$
$$2x_1 + 2x_2 + 2x_3 \geq 0$$
(b) $x_1 \in \{2,3,6,11\}$ (7%)
(c) $x_1 = 0$ or $x_1 \geq 100$ (6%)

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