

國立臺灣科技大學
九十學年度碩士班招生考試試題

系所組別：資訊管理系甲組

科目：離散數學

1. (12 points) Let $P(x) = x^4 + x + 1$ be a polynomial with coefficients from $\text{GF}(2)$, where GF denotes the Galois field. Please check in detail whether $P(x)$ is irreducible or not.
2. (13 points) Given two events A and B , let the mutual information be $I(A, B) = -\log p(A) + \log P(A|B)$, where $P(A)$ denotes the probability of A and $P(A|B)$ denotes the conditional probability of event A given the occurrence of event B . Show that $I(A, B) = I(B, A)$.
3. (12 points) Show that the lower bound of the sorting problem is $\Omega(n \log n)$ by using the comparison tree model.
4. (13 points) Show that the set of positive rational numbers is countable.



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5. (10 points) The adjacency matrix A of a simple graph G is as follows:

	a	b	c	d	e
a	0	1	0	1	0
b	1	0	1	0	1
A= c	0	1	0	1	1
d	1	0	1	0	0
e	0	1	1	0	0

How many paths of length 4 are there from node d to node e ?

6. (15 points) Solve the following recurrence relation

$$a_n = 5a_{n-1} - 6a_{n-2},$$

with the initial conditions $a_0 = 7$, $a_1 = 16$.

7. (15 points) Find the time complexity of the following algorithm. (i.e., compute the number of times the statement $x := x + 1$ is executed.)

```

1. j := n
2. while j ≥ 1 do
3.   begin
4.     for i := 1 to j do
5.       x := x + 1;
6.     j := ⌊j/2⌋
7.   end

```

8. (10 points) A relation R defined on the set of eight-bit strings by $s_1 R s_2$ provided that the first four bits of s_1 and s_2 coincide. Examples:

00101101R00100001

- Show that R is an equivalence relation.
- How many equivalence classes are there?



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