

國立臺灣科技大學

九十一學年度碩士班招生考試試題

系所組別：機械工程系甲組、機械工程系乙組、機械工程系丙組、機械工程系丁組、機械工程系戊組  
 科目：工程數學

※總分：100分

1. Solve the ordinary differential equation

$$y'' + 4y = x \cos 2x \quad (20\%)$$

2. (a) Find the constant
- $k$
- so that the vector field
- $xy\vec{i} + kx^2\vec{j}$
- is conservative, and then find the potential function of the vector field. (10%)

- (b) Calculate
- $\iiint_S \mathbf{x}\vec{R} \cdot \vec{N} dA$
- in terms of the volume
- $V$
- and the
- $x$
- coordinate of the centroid of the volume
- $\bar{x}$
- , where
- $S$
- is a regular, closed surface bounding the volume
- $V$
- ;
- $\vec{R}$
- is the position vector, and
- $\vec{N}$
- is the outer normal vector of
- $S$
- . (10%)

3. Use an orthogonal transformation to diagonalize the matrix

$$[A] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix},$$

i.e.,  $[Q] = [P]^{-1}[A][P]$  is a diagonal matrix, in which  $[P]$  is an orthogonal matrix.Write down the matrices  $[Q]$  and  $[P]$ . (20%)

4. Solve the following boundary-value problem by the method of separation of variables

$$\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty, \quad t \geq 0$$

$$u(x, 0) = 0, \quad \frac{\partial u}{\partial t}(x, 0) = \delta(x) \quad (\delta(x) \text{ is the Dirac-delta function}) \quad (20\%)$$

5. Evaluate the integral
- $\int_{-\infty}^{\infty} \frac{\sin x}{x} dx$
- by the residue theorem. (20%)

