國立臺灣科技大學
九十一學年度碩士班招生考試試題

系所組別：機械工程系丙組
科　　目：流體力學

※總分：100分

Note: All the mathematical relations and governing equations needed are listed in page 3

1. Write down or expand the following terms for Spherical Coordinates: 每題6分，共12分
   (a) θ component for the acceleration ($\vec{a} = D\vec{v}/Dt$),
   (b) Definition of stream function for a compressible flow.

2. Regarding Reynolds transport theory: 每題6分，共18分
   (a) What are the benefits brought by this theory?
   (b) By means of this theory, please derive the definition for $div\vec{v}$
   (c) From this definition and continuity equation, please explain the difference between $div\vec{v}$ and $\rho=$ constant.

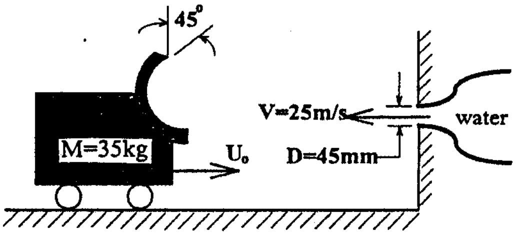
3. Consider two-dimensional laminar boundary-layer flow along a flat plate. The velocity profile is assumed to be 每題6分，共18分

$$\frac{u}{U} = 2\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$$

   Find expressions for:
   (a) The rate of growth of δ as a function of x (Re_x),
   (b) The displacement thickness, δ^*, as a function of x (Re_x),
   (c) The friction coefficient, C_f, as a function of x (Re_x).

4. Water flows in a circular pipe. At one section the diameter is 0.3 m, the static pressure is 300 kPa(gage), the velocity is 5 m/s, and the elevation is 20 m above ground level. At a section downstream at 10 m above the ground level, the diameter is 0.25 m. Find the gage pressure at the downstream section if frictional effect can be neglected. 12分

5. A curved vane and cart assembly moves horizontally toward a water jet, under the conditions shown in below. The mass of the assembly is $M=35kg$ and its initial speed is $U_o=6m/s$. Neglect rolling resistance and aerodynamic drag. Evaluate the time and distance needed for the cart assembly to slow to a stop. 12分



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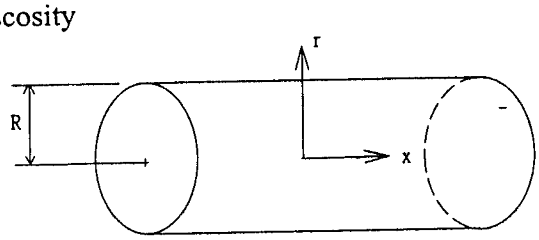
6. By means of the following assumptions 每小題7分；本大題共28分

(1) Treat it as 2-D ($\partial/\partial z = 0$)
(2) Incompressible flow
(3) Steady state ($\partial/\partial t = 0$)
(4) $V_r = V_z = 0$ and $V_\theta = V_\theta(r)$, $P = P(r)$ only.
(5) Neglect body force
(6) Constant viscosity



Please derive the following items for pipe flow

(a) Expand and simplify the continuity equation,
(b) The expression of Vorticity $\vec{\Omega} = \Omega_x \hat{e}_x + \Omega_y \hat{e}_y + \Omega_z \hat{e}_z$,
(c) Simplify $div\,\bar{\tau}$ to

$$div\,\bar{\tau} = \frac{d\Omega_z}{dr}\hat{e}_\theta ,$$

(d) Finally, you should obtain:

$$\frac{d^2V_\theta}{dr^2} + \frac{d}{dr}\left(\frac{V_\theta}{r}\right) = 0 \qquad \text{and} \qquad \rho\frac{V_\theta^2}{r} = \frac{dP}{dr}$$

Please specify the boundary condition for these equations.

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Summary of Equations

(1) Continuity Equation:

$$0=\frac{\partial}{\partial t}\int_{CV}\rho d\forall+\int_{CS}\rho\vec{V}\cdot d\vec{A}$$

$$\nabla\cdot\rho\vec{V}+\frac{\partial\rho}{\partial t}=0 \quad \text{or} \quad \frac{D\rho}{Dt}+\rho div\vec{V}=0$$

(2) Momentum Equation:

$$\vec{F}_S+\vec{F}_B-\int_{CV}\left[\vec{a}_{rf}+2\vec{\omega}\times\vec{V}_{xyz}+\vec{\omega}\times(\vec{\omega}\times\vec{r})+\dot{\vec{\omega}}\times\vec{r}\right]\rho d\forall$$

$$=\frac{\partial}{\partial t}\int_{CV}\vec{V}_{xyz}\rho d\forall+\int_{CS}\vec{V}_{xyz}\rho\vec{V}_{xyz}\cdot d\vec{A}$$

$$\rho\frac{D\vec{V}}{Dt}=-\nabla p+div\,\vec{\tau}+\rho\vec{g}$$

(3) The First Law of Thermodynamics:

$$\dot{Q}-\dot{W}_s-\dot{W}_{\text{shear}}-\dot{W}_{\text{other}}=\frac{\partial}{\partial t}\int_{CV}e\rho d\forall+\int_{CS}\left(u+pv+\frac{V^2}{2}+gz\right)\rho\vec{V}\cdot d\vec{A}$$

(4) Mathematics relations:

(a) Gradient Theorem

$$\iiint_V\nabla\phi d\tau=\oiint_S\phi\hat{n}\,dS$$

(b) Divergence Theorem

$$\iiint_V \text{div}\,\vec{A}d\tau=\oiint_S\vec{A}\cdot\hat{n}dS$$

(c) Curl Theorem

$$\iiint_V\text{curl}\vec{A}d\tau=\oiint_S\hat{n}\times\vec{A}dS$$

(d) Gradient

$$\nabla\phi=\frac{\hat{e}_1}{h_1}\frac{\partial\phi}{\partial q^1}+\frac{\hat{e}_2}{h_2}\frac{\partial\phi}{\partial q^2}+\frac{\hat{e}_3}{h_3}\frac{\partial\phi}{\partial q^3}$$

(e) $$\text{div}\,\vec{A}=\frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial q^1}\left(h_2h_3\hat{A}_1\right)+\frac{\partial}{\partial q^2}\left(h_1h_3\hat{A}_2\right)+\frac{\partial}{\partial q^3}\left(h_1h_2\hat{A}_3\right)\right]$$

(f) $$\text{curl}\,\vec{A}=\frac{1}{h_1h_2h_3}\begin{vmatrix}h_1\hat{e}_1 & h_2\hat{e}_2 & h_3\hat{e}_3\\ \frac{\partial}{\partial q^1} & \frac{\partial}{\partial q^2} & \frac{\partial}{\partial q^3}\\ h_1\hat{A}_1 & h_2\hat{A}_2 & h_3\hat{A}_3\end{vmatrix}$$

(g) $$\vec{V}\cdot\nabla\vec{V}=\nabla\left(\frac{\vec{V}^2}{2}\right)-\vec{V}\times\text{curl}\vec{V}$$

(h) $$div\vec{\tau}=\mu\nabla^2\vec{V}+(\lambda+\mu)grad\left(div\vec{V}\right)+2grad\mu\cdot\vec{\varepsilon}+\left(div\vec{V}\right)grad\lambda$$

(i) $$\nabla^2\vec{V}=grad\left(div\vec{V}\right)-curl\,curl\,\vec{V}$$