國立臺灣科技大學
九十一學年度碩士班招生考試試題

系所組別：化學工程系
科　目：工程數學

總分一百分。可不依序作答，但請標明題號。務必詳列計算過程，否則酌予扣分。

1. Solve the differential equation: $y' + y\tan x = \sin 2x$; $y(0) = 1$ (10%)

2. Solve the differential equation: $y'' + 2y = f(t)$,
where $f(t) = 1, 0 \le t < 1$; $f(t) = 0, t \ge 1$; $y(0) = y'(0) = 0$ (15%)

3. Solve the differential equation: $x^2y'' - 4xy' + 6y = 2x^4 + x^2$ (15%)

4. Find all the eigenvalues and the eigenvector associated with the smallest eigenvalue of the following matrix.

$$\begin{bmatrix} 0 & 1 & -2 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$$ (10%)

5. $f(t)$ is a periodic function with period 2.
$f(t) = \pi t$, $-1 < t < 1$
(a) Expand $f(t)$ as a Fourier series (10%)
(b) Solve the 2nd order differential equation

$$\frac{1}{16}\frac{d^2y}{dt^2} + 4y = f(t)$$ (10%)

6. Solve the 2nd order ordinary differential equation

$$2x^2\frac{d^2y}{dx^2} + x(1+4x)\frac{dy}{dx} + (2x-1)y = 0$$ (15%)

7. Solve the partial differential equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0 \quad (0 < x < a, 0 < y < b)$$

with the corresponding boundary conditions
$f(x, 0) = f(x, b) = 0$ $(0 < x < a)$
$f(0, y) = 0$ $f(a, y) = A = \text{constant}$ $(0 < y < b)$ (15%)