

國立臺灣科技大學

九十一學年度碩士班招生考試試題

系所組別：資訊工程系、電子工程系甲組

科目：離散數學

1. 20%

- (a) 5% Derive the explicit form for the Fibonacci sequence a_n in terms of n which satisfies the following recursive formula: $a_1 = a_2 = 1$ and $a_k = a_{k-1} + a_{k-2}$, where $k \geq 3$.
- (b) 6% Consider the well-known Tower of Hanoi problem which consists of three pegs with disks placed on one of the pegs, say peg A , in descending order. Let $H(k)$ represent the number of moves to transfer k disks from one peg to another.
- 2% Write down the recursive formula for $H(k)$ and $H(k-1)$.
 - 4% Derive the explicit expression in terms of n for $H(n)$.
- (c) 5% Solve the following sets of simultaneous congruences:

$$\begin{aligned} x &\equiv 5 \pmod{6} \\ x &\equiv 3 \pmod{10} \\ x &\equiv 8 \pmod{15} \end{aligned}$$

- (d) 4% A box contains 3 white balls and 2 black balls. Two balls are drawn from it without replacement. Calculate the probability that the first ball is white given that the second ball is white.

2. 15%

- (a) 4% Given the adjacency matrix (assume that a, b, c, d, e, f represent the vertices)

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- 1% Is the graph connected?
 - 1% Is the graph strongly connected?
 - 1% Does the graph have an Euler cycle?
 - 1% Is there a 3-path from a to f ?
- (b) 6% If a tree T has e edges and v vertices, then
- 1% Write down the relation between e and v .
 - 5% Prove the above result using induction.
- (c) 5% Prove that if for any two vertices in a graph G , there is a unique path, then G is a tree.



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3. 15%

- (a) 9% Let $A = \{1, 2, 3, 4, 5\}$, $B = \{6, 7, 8, 9\}$, $C = \{10, 11, 12, 13\}$, and $D = \{\alpha, \beta, \gamma, \delta\}$. Also let $R \subseteq A \times B$, $S \subseteq B \times C$, and $T \subseteq C \times D$ be defined by $R = \{(1, 7), (4, 6), (5, 6), (2, 8)\}$, $S = \{(6, 10), (6, 11), (7, 10), (8, 13)\}$, and $T = \{(11, \beta), (10, \beta), (13, \delta), (12, \alpha), (13, \gamma)\}$. Compute the relations:
- 3% R^{-1} and S^{-1} .
 - 3% $T \circ (S \circ R)$
 - 3% Describe the smallest reflexive relation on D .
- (b) 6% Determine whether the relation R is a partial ordering on $E = \{a, b, c, d\}$ or not.
- 3% E is the set of all people and R is defined by xRy if x is order than y .
 - 3% E is the set of all integers and R is defined by xRy if $x \geq 2y$.

4. 17% Prove that 3-SATISFIABILITY is NP-complete.

5. 17% Let $(G, *)$ be a group and let S be a subset of G . If S is a finite set, then prove that $(S, *)$ is a subgroup of $(G, *)$ if $*$ is a closed operation on S .

6. 16% Before performing the heapsort on the tree, we need heapify the tree. Please analyze the time complexity in detail for heapifying the tree.

