

## 國立臺灣科技大學

## 九十一學年度碩士班招生考試試題

系所組別：電機工程系甲組、電機工程系乙二組

科目：工程數學

(共六題；滿分 100 分)

1. Let  $\mathbf{F} = (yze^{xyz} - 4x)\hat{a}_x + (xze^{xyz} + z)\hat{a}_y + (xye^{xyz} + y)\hat{a}_z$  for all  $x, y$  and  $z$ .(a) Verify that  $\mathbf{F}$  is conservative. (5%)(b) Find a potential function for  $\mathbf{F}$ . (10%)2. Let  $g$  be a periodic function defined by

$$g(t) = t^2 \text{ for } 0 < t < 3 \text{ and } g(t+3) = g(t) \text{ for all } t.$$

(a) Draw the graph of  $g$  for  $-6 < t < 6$ . (5%)(b) Compute the Fourier series of  $g$ . (10%)(c) Draw the amplitude spectrum of  $g$  for the three lowest-frequency components. (5%)3. Evaluate  $\oint_C 1/(1+z^2) dz$  if  $C$  is any piecewise-smooth simple closed curve in the complex plane.Consider all possible cases, which do not pass through  $i$  or  $-i$ . (15%)4. Find the general solution  $y(x)$  to

$$y'' - 8y' + 16y = 8\sin(2x) + 3e^{4x}. \quad (15\%)$$

5. Solve the initial value problem for  $y(t)$  with Laplace transform:

$$y'' + 2ty' - 4y = 1; \quad y(0) = y'(0) = 0. \quad (10\%)$$

6. Use the matrix exponential to solve the following initial value problems:

$$\frac{d}{dt} Y(t) = AY(t), \quad Y(0) = Y_0.$$

$$(1) \quad A = \begin{pmatrix} 3 & 4 \\ 3 & 2 \end{pmatrix}, \quad Y_0 = \begin{pmatrix} 6 \\ 1 \end{pmatrix}, \quad \text{and} \quad Y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} \quad (15\%)$$

$$(2) \quad A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_0 = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}, \quad \text{and} \quad Y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{pmatrix} \quad (10\%)$$

